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Question Paper Code:U4M21

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Fourth Semester

Computer Science and Engineering

21UMA421 - TRANSFORMS AND DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- $P \rightarrow \neg Q$ is equivalent to CO6-U
(a) $\neg P \wedge Q$ (b) $P \wedge \neg Q$ (c) $\neg(P \wedge Q)$ (d) $P \vee \neg Q$
- How many “F” are occurred in $(P \rightarrow \neg Q)$ CO1-U
(a) 4 (b) 3 (c) 1 (d) 2
- If a bit string contains $\{0, 1\}$ only, having length 5 has no more than 2 ones in it. Then calculate how many such bit strings are possible? CO2- App
(a)14 (b)12 (c)16 (d)12
- Calculate how many integers between 1 to 100 are divisible by 2 and 6 CO2- App
(a) 41 (b)16 (c)82 (d) 26
- The inverse of the element [3] in a group (Z_5, \oplus_5) CO3- App
(a) [2] (b)[4] (c)[3] (d) [1]
- The union of two subgroup of G is a CO6- U
(a) Subgroup (b) semigroup (c) group (d) Monoid
- Fourier Sine transform of e^{-5x} CO4- App
(a) $\sqrt{\frac{2}{\pi}} \frac{5}{s^2 + 25}$ (b) $\sqrt{\frac{2}{\pi}} \frac{s}{s^2 - 25}$ (c) $\sqrt{\frac{2}{\pi}} \frac{s}{s^2 + 25}$ (d) $\sqrt{\frac{2}{\pi}} \frac{5}{s^2 - 25}$

8. Fourier cosine transform of $1, 0 < x < 1$ CO4-App
- (a) $\frac{\cos s}{s}$ (b) $\sqrt{2\pi} \frac{\cos s}{s}$ (c) $\sqrt{\frac{2}{\pi}} \frac{\sin s}{s}$ (d) None of these

9. The Z transform of 2^n is CO6-U
- (a) $\frac{z}{z-1}$ (b) $\frac{2z}{(z-1)^2}$ (c) $\frac{z}{(z-1)^2}$ (d) $\frac{z}{(z-1)^3}$

10. The Z^{-1} transform of $\frac{z}{(z-4)^2}$ is CO5-U
- (a) 4^n (b) $(-4)^n$ (c) $n(-4)^{n-1}$ (d) $n(4)^{n-1}$

PART – B (5 x 2= 10Marks)

11. Compute PDF for $(P \vee Q)$ CO1-App
12. In how many ways can letters of the word “THUNAIEZHUTHU” be arranged CO2- App
13. For a Group $G = \{1, -1, -i, i\}$ under multiplication ,Find order of all elements CO3- App
14. Find the Fourier sine transform of $e^{-ax}, a > 0$ CO4- App
15. Determine the Z – Transform of $z(n)$ CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Calculate PCNF and PDF for $(P \wedge \neg Q) \vee (P \wedge R) \vee (Q \wedge R)$ CO1-App (8)
- (ii) Using the rules of inference derive & using CP Rule. CO1-App (8)
- $P \rightarrow (Q \rightarrow S), \neg R \vee P, Q \Rightarrow R \rightarrow S$
- Or
- (b) (i) Prove the following by Indirect Method. CO1-App (8)
- $P \rightarrow Q, Q \rightarrow R, \neg(P \wedge R), P \vee R \Rightarrow R$
- (ii) Prove the following by direct method CO1-App (8)
- $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$
17. (a) (i) Solve $a_n - 6a_{n-1} + 9a_{n-2} = 3^n, a_0 = 1, a_1 = 1$ CO2- App (8)
- (ii) Calculate the number of positive integers not exceeding 1300 that are divisible by 2, 3, 5 or by 7 CO2- App (8)

Or

(b) (i) Using mathematical induction CO2- App (8)

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)} = \frac{n}{(n+1)}$$

(ii) Using generating functions Solve $a_n = 2a_{n-1} + 4^n, a_0 = 1$ CO2- App (8)

18. (a) State and prove Fundamental theorem of homomorphism CO3- U (16)

Or

(b) (i) The necessary and sufficient condition that a non empty subset of a group G be a Subgroup is $a \in H, b \in H \Rightarrow a * b^{-1} \in H$ CO3- U (8)

(ii) A group G is abelian iff $(a * b)^2 = a^2 * b^2$ CO3- U (8)

19. (a) Compute the Fourier Transform of CO4 App (16)

$$f(x) = \begin{cases} a - |x| & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases} \quad \text{and hence evaluate}$$

(i) $\int_0^{\infty} \left(\frac{\sin x}{x}\right)^4 dx$ (ii) $\int_0^{\infty} \left(\frac{\sin x}{x}\right)^2 dx$

Or

(b) Evaluate CO4 App (16)

(i) $\int_0^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 4)}$

(ii) $\int_0^{\infty} \frac{x^2 dx}{(x^2 + 9)^2}$ using Fourier transform

20. (a) (i) Solve the difference equation $y_{n+2} - 6y_{n+1} + 5y_n = 3^n$ given CO5- App (8)

that $y_0 = 0, y_1 = 0$

(ii) Using Convolution theorem find CO5- App (8)

$$Z^{-1} \left[\frac{16z^2}{(8z-5)(2z+1)} \right]$$

Or

(b) (i) Solve the difference equation $y_{n+2} + 3y_{n+1} - 10y_n = 8^n$ given CO5- App (8)

that $y_0 = 0, y_1 = 0$

(ii) Using Convolution theorem find CO5- App (8)

$$Z^{-1} \left[\frac{20z^2}{(5z-3)(4z+3)} \right]$$

