A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U3023

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Third Semester

Electronics and Communication Engineering

21UMA323- NUMERICAL ANALYSIS AND LINEAR ALGEBRA

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. Trapezoidal rule is so called, because it approximates the integral by the sum of _______trapezoids CO1 -U

   (a) n (b) n+1 (c) n-1 (d) 2n

2. In Simpson's 3/8 rule the number of subintervals should be________ CO1 -U

   (a) multiple of 1 (b) multiple of 2 (c) multiple of 3 (d) All of these

3. In Euler's method, if h is large then it gives _______value CO2 -U

   (a) accurate (b) inaccurate (c) average (d) None of these

4. The Runge-Kutta method of second order is nothing but the ___________ CO2 -U

   (a) Euler's method (b) modified Euler's method

   (c) improved Euler's method (d) Taylor's series method

5. PDE of second order, if B^2-4AC< 0 then CO6- U

   (a) parabolic (b)elliptic (a) hyperbolic (b)cyclic

6. Bender-Schmidt recurrence equation is valid if λ= CO3- U

   (a) $\frac{1}{2}$ (b)$\frac{1}{3}$ (C) 3/2 (d) 2/3

7. In a vector space V ,for every $x, y \in V$ then the property $x + y = y + x$ is known as _________ CO4- U

   (a) commutative (b) associative (c) identity (d) inverse

8. If T: V→W be linear transformation then T(0) = _________ CO4- U

(a) 0 (b) 1 (c) 2 (d) 3

9. In a vector sapace V , for $x, y, z \in V$ if $\langle x, y \rangle = \langle y, z \rangle$ then _________ CO5- U

(a) $y = z$ (b) $y \neq z$ (c) $y = -z$ (d) none of these

10. For any two vectors x and y in an inner product space V , , $\|x + y\| \leq$ _________ CO5- U

(a) $\|x\| + \|y\|$ (b) $\|x\|\|y\|$ (c) $\|x\| - \|y\|$ (d) $\|x\| / \|y\|$

PART – B (5 x 2= 10Marks)

11. Using two –point Gaussian quadrature formula evaluate $\int_{-1}^{1} (3x^2 + 5x^4)dx$ CO1- App

12. Using Euler's method find y(0.2) given $\frac{dy}{dx} = y + e^x$, y(0) =0 CO2- App

13. Classify $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ CO3- App

14. If W_1, W_2 are two subspaces of a finite dimensional vector space V with $\dim W_1 = 2$, $\dim W_2 = 3$, $\dim(W_1 + W_2) = 4$ then find $\dim(W_1 \cap W_2)$ CO4- App

15. Find the norm of $(1,2,3)$ in $V_3(R)$ with standard inner product. CO5 -App

PART – C (5 x 16= 80Marks)

16. (a) (i) Compute $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ at x=1.5 from the following data CO1- App (8)

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.000	13.625	24.000	38.875	59.000

(ii) Evaluate $\int_{0}^{\pi} \sin x dx$ by dividing the range into 10 equal parts using (i) Trapezoidal rule (ii) Simpson's $\frac{1}{3}$ rule. CO1- App (8)

Or

	(b)	(i) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg's method correct to 4 decimal places.	CO1- App	(8)
		(ii) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using two point Gaussian quadrature formula.	CO1- App	(8)
17.	(a)	(i) Using Taylor's series method find y(1.1) given $y' = x + y$ with y(1)=0	CO2- App	(8)
		(ii) Using Euler's method find y(0.1) and y(0.2) from $y' = 1 - y$, y(0)=0	CO2- App	(8)
		Or		
	(b)	(i) Using R-K method of fourth order, find y(0.1) for the initial value problem $\frac{dy}{dx} = x + y^2$, y(0) = 1	CO2- App	(8)
		(ii) Given $\frac{dy}{dx} = x^3 + y$, y(0) = 2 , y(0.2) = 2.443 , y(0.4) = 2.99 y(0.6) = 3.68 . Calculate y(0.8) by Milne's Predictor & Corrector method.	CO2- App	(8)
18.	(a)	(i) Solve $\frac{\partial^2 u}{\partial x^2} = 32\frac{\partial u}{\partial t}$, u(0,t) = 0 , u(1,t) = t , u(x,0) = 0 . Take h = 0.25 and find the values of u up to t = 5 using Bender-Schmidt's difference equation.	CO3- App	(8)
		(ii) Using Crank-Nicholson's difference equation to solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ u(0,t) = 0 , u(1,t) = t , u(x,0) = 0. compute u for one time step function with h=0.25 .	CO3- App	(8)
		Or		
	(b)	Solve the Poisson equation $u_{xx} + u_{yy} = -81xy$, 0<x<1, 0<y<1 , u(0,y)=0, u(x,0) = 0, u(1,y)=100, u(x,1)=100 and h=1/3	CO3- App	(16)

19. (a)	(i) Verify the vectors (1,2,-3) , (2,5,1) ,(-1,1,4) in $\boldsymbol{R}^3$ is a basis of $\boldsymbol{R}^3$	CO4- App	(8)
	(ii) If $\boldsymbol{T}:\boldsymbol{R}^2\to\boldsymbol{R}^3$ be linear transformation defined by $\boldsymbol{T}(\boldsymbol{a}_1,\boldsymbol{a}_2)=(\boldsymbol{a}_1-\boldsymbol{a}_2,0,0)$ then find nullity(T) ,rank(T), Is T one-to-one? Is T onto? Also check the rank nullity theorem.	CO4- App	(8)
	Or		
(b)	(i) Find the dimension of the subspace spanned by the vectors (2,0,1) ,(-1,0,1) ,(1,0,2) in $V_3(\boldsymbol{R})$	CO4- App	(8)
	(ii) Find the matrix of the linear transformation $\boldsymbol{T}:\boldsymbol{V}_2(\boldsymbol{R})\to\boldsymbol{V}_3(\boldsymbol{R})$ defined by $\boldsymbol{T}(\boldsymbol{a},\boldsymbol{b})=(\boldsymbol{a}-\boldsymbol{b},2\boldsymbol{a},3\boldsymbol{a}+2\boldsymbol{b})$ for the standard basis of $\boldsymbol{V}_2(\boldsymbol{R})$.	CO4- App	(8)
20. (a)	(i) Show that the following function defines an inner product on $\boldsymbol{V}_2(\boldsymbol{R})$ where $\boldsymbol{x}=(\boldsymbol{x}_1,\boldsymbol{x}_2)$ and $\boldsymbol{y}=(\boldsymbol{y}_1,\boldsymbol{y}_2)$ and $\langle\boldsymbol{x},\boldsymbol{y}\rangle=6\boldsymbol{x}_1\boldsymbol{y}_1+7\boldsymbol{x}_2\boldsymbol{y}_2$	CO5- App	(8)
	(ii) If $\boldsymbol{x}=(2,1+\boldsymbol{i},\boldsymbol{i})$ and $\boldsymbol{y}=(2-\boldsymbol{i},2,1+\boldsymbol{i})$ then verify Schwarz's inequality.	CO5- App	(8)
	Or		
(b)	Apply Gram-Schmidt process to construct an orthonormal basis for $\boldsymbol{V}_3(\boldsymbol{R})$ with the standard inner product for the basis $\{\boldsymbol{v}_1,\boldsymbol{v}_2,\boldsymbol{v}_3\}$ where $\boldsymbol{v}_1=(1,-1,0)$, $\boldsymbol{v}_2=(2,-1,-2)$ and $\boldsymbol{v}_3=(1,-1,2)$	CO5- App	(16)