A **Reg. No. :** | | | | | | | | | | |

Question Paper Code: 53021

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Third Semester

Civil Engineering

15UMA321- Transforms and Partial Differential Equations

(Common to MECH, ECE, EEE,EIE, CHEM, AGRI, BME)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Identify b_n in the expansion of x^2 as a Fourier series in $(-\pi, \pi)$. CO1- R

(a) 0 (b) 1 (c) -1 (d) 2

2. If f(x) is an odd function defined in $(-\ell,\ell)$, list the values of a_0 and a_n ? CO1- R

(a) $a_0 = 1, a_n = 1$ (b) $a_0 = 0, a_n = 0$ (c) $a_0 = 0, a_n = 1$ (d) $a_0 = 1, a_n = 0$

3. Identify the Fourier Cosine transform of e^{-x} CO2- R

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{1}{1+s^2}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{1}{a^2+s^2}\right]$ (c) $\sqrt{\frac{\pi}{2}}\left[\frac{1}{1+s^2}\right]$ (d) $\sqrt{\frac{\pi}{2}}\left[\frac{1}{a^2+s^2}\right]$

4. Examine a function which is self reciprocal under Fourier Transform. CO2- R

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $-\frac{1}{x}$ (d) $\frac{1}{\sqrt{x}}$

5. Identify Z $[a^n]$ CO3- R

(a) $\frac{z}{z-a}$ (b) $\frac{z}{z+a}$ (c) $\frac{z-a}{z}$ (d) $\frac{z+a}{z}$

6. Identify $Z[\sin\frac{n\pi}{2}]$. CO3- R

(a) $\frac{z}{z^2-1}$ (b) $\frac{-z}{z^2+1}$ (c) $\frac{-z}{z^2-1}$ (d) $\frac{z}{z^2+1}$

7. Identify the complementary function of $(D^2 - 4DD' + 3D'^2)Z = 0.$ CO4- R

(a) $\phi_1(y+x) + \phi_2(y+3x)$ (b) $\phi_1(y+x) - \phi_2(y+3x)$

(c) $\phi_2(y+2x) + \phi_1(y+x)$ (d) $\phi_1(y-x) - \phi_2(y+3x)$

8. Examine the complete integral of $z = px + qy + p^2q^2$. CO4- R

(a) $z = ax + by + a^2b^2$ (b) $z = ax - by - a^2b^2$

(c) $z = ax - by + a^2b^2$ (d) $z = ax - by - a^2$

9. Identify the partial differential equation of 3uxx+4uxy+3uy -2ux = 0. CO5- R

(a) Elliptic (b) Parabolic (c) Hyperbolic (d) None of these

10. The ends A & B of a rod of length 10cm have their temperature kept at $20^\circ C$ and $70^\circ C$. Examine the steady state temperature distribution on the rod. CO5- R

(a) 5x+ 20 (b) 7x+ 10 (c) 2x+ 20 (d) 7x- 10

PART – B (5 x 2= 10 Marks)

11. List the Dirichlet's conditions on Fourier series. CO1- R

12. If F(s) is the Fourier Transform of f(x). Identify F[f(x-a)]= e^{ias} F(s). CO2- R

13. Identify $z(n) = \frac{z}{(z-1)^2}$, $|z| > 1$. CO3- R

14. Identify the difference equation by eliminating arbitrary constants for, CO4- R

y =$A2^n + Bn$.

15. List the three possible solutions for Two dimensional heat equation. CO5- R

PART – C (5 x 16= 80Marks)

16. (a) (i) Illustrate the Fourier Series of CO1- App (8)

$$f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$$ and deduce

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots + \infty = \frac{\pi^2}{8}.$$

(ii) Apply the Fourier Series of $f(x) = 2x - x^2$ in the interval 0< x < 3. CO1- App (8)

Or

(b) (i) Illustrate the cosine series for the function CO1- App (8)

$f(x) = x(\pi - x)$ in $(0, \pi)$ and hence deduce that

$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$

(ii) Solve the first two harmonic of the Fourier series from the following data CO1- App (8)

X	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
Y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

17. (a) (i) Illustrate the Fourier Transform of f(x) if $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence deduce the value of CO2- App (8)

(1) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$

(2) $\int_0^\infty \left(\frac{\sin^4 t}{t^4}\right) dt$

(ii) Show that CO2- App (8)

$e^{\frac{-x^2}{2}}$ is self -reciprocal under Fourier Transform.

Or

(b) (i) Analyze CO2- App (8)

$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier Transforms.

(ii) Analyze CO2- App (8)

$\int_0^\infty \frac{dx}{(a^2+x^2)^2}$ using transform method .

18. (a) (i) Infer $Z[r^n \cos n\theta]$ and $Z[r^n \sin n\theta]$. CO3- Ana (8)

(ii) Analyze inverse z transform of CO3- Ana (8)

$\frac{8z^2}{(2z-1)(4z-1)}$ using Convolution theorem.

Or

(b) (i) If CO3- Ana (8)

$U(z) = \frac{2z^2+5z+14}{(z-1)^2}$, point out u_2 and u_3.

(ii) Analyze the equation $y(n+3) - 3y(n+1) + 2y(n) = 0$ given that y(0)= 4, y(1)=0, y(2)=8. CO3- Ana (8)

19. (a) (i) Interpret the PDE by eliminating the arbitrary functions f and φ from $z = xf\left(\frac{y}{x}\right) + y\,\phi(x)$. CO4- Ana (8)

(ii) Interpret $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. CO4- Ana (8)

Or

(b) (i) Analyze $z = px + qy + p^2 - q^2$. CO4- Ana (8)

(ii) Analyze $\left(D^3 - 7DD'^2 - 6D'^3\right)Z = \sin(x+2y) + e^{3x+y}$. CO4- Ana (8)

20. (a) A string is stretched and fastened to points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Interpret the displacement of any point on the string at a distance of x from one end at time t. CO5- U (16)

Or

(b) An infinitely long rectangular plate with insulated surfaces is 10cm wide. The two long edges and one short edge are kept at 0˚C, while the other short edge x = □ 0 is kept at temperature $u = \begin{cases} 20y, & 0 \le y \le 5 \\ 20(10-y), & 5 \le y \le 10 \end{cases}$. Interpret the steady state temperature at any point in the plate. CO5- U (16)

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