A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M03

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Second Semester

Computer Science and Engineering

21UMA203- Differential Equations and Complex analysis

(Regulations 2021)

(Common to information technology)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{1}{(D-m)^2}e^{mx}$ = _____ CO1-Ap

(a)) xe^{mx} (b)x^2e^{mx} (c) $\frac{x^2}{2}e^{mx}$ (d) $\frac{x^2}{m}e^{mx}$

2. The complementary function of $(4D^2- 3D-1)y=2\sin 2x$ is _____ CO6-U

(a) $Ae^x + Be^{-\frac{x}{4}}$ (b) $Ae^{-x}+Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^{x}+Be^{4x}$

3. Div $\bar{r}$ = ____ CO2-App

(a) 0 (b)1 (c)3 (d) $\bar{r}$

4. Divergence of vector $x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ at (1, 2, -3) is _____ CO2-App

(a) 8 (b)4 (c)-3 (d) 0

5. The critical point of the transformation $w = z + \frac{1}{z}$ are ____ CO3- App

a) ± 1 b) ± 2 c) $\pm i$ d) $-i$

6. The function $f(z) = \frac{1}{z^2+4}$ is not analytic at z = ____ . CO3- App

(a) 2 b) -2 c)2i d)$\pm 2i$

7. Simple pole is a pole of order _____ CO6-U

(a) 1 (b) 4 (c) 3 (d) -4

8. $\int_C \frac{e^z}{z-2} dz$ where C is the unit circle with centre as origin is _____ CO4-App

(a) 0 (d) 1 (c) 2 (d) π

9. The PDE obtained from z = (x+a)(y+b) is __. CO5-App

(a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

10. The subsidiary equations of Lagrange's linear equation is ---------- CO5-U

(a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ (b) $\frac{dx}{P} + \frac{dy}{Q} + \frac{dz}{R}$ (c) $\frac{dx}{P} - \frac{dy}{Q} - \frac{dz}{R}$ (d) Pp +Qq= R

PART – B (5 x 2= 10Marks)

11. Find the Wronskian of y_1 ,y_2 of y" -2y' +y =e^x logx CO1-App

12. Compute $\nabla\varphi$, if $\varphi = x^2 + y^2 + z^2$ at (1, -1, 1). CO2-App

13. Prove that $u = e^x \cos y$ is harmonic function CO3-App

14. Using Cauchy's integral formula , Evaluate $\int_c \frac{z}{z-2} dz$ where C is $|z| = 1$ CO4-App

15. Find the particular integral of $(D^2 - 2DD' + D'^2)Z = \cos(x - 3y)$ CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve $(D^2 + 2D + 2)y = e^{-2x} + \cos 2x$ CO1-App (8)

(ii) Using method of variation of parameters solve $(D^2 + a^2)y = \text{Cosec } ax$ CO1- App (8)

Or

(b) (i) Solve $(x^2D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$ CO1- App (8)

(ii) A colony of bacteria of growing exponentially. At time t=0 it has 10 bacteria in it and at time t = 4 it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)

17. (a) Verify Green's theorem in the XY plane for $\int_C (3x^2 - 8y2dx+4y-6xydy$ where C is the boundary of the region defined by $x = y^2, y = x^2$. CO2-App (16)

Or

(b) Verify Gauss divergence theorem for the vector function $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ $and x = 1, y = 1, z = 1$ — CO2 -App — (16)

18. (a) (i) Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ — CO3-App — (8)

(ii) Determine the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$ — CO3-App — (8)

Or

(b) (i) If f(z)=u+iv is an analytic function then Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f^1(z)|^2$ — CO3-App — (8)

(ii) Determine the bilinear transformation which maps z = 1,i,-1 respectively onto w = i,0,-i — CO3-App — (8)

19. (a) (i) Using Cauchy's integral formula , Evaluate $\int_c \frac{z+1}{(z-3)(z-1)} dz$ where C is the circle $|z| = 2$ — CO4-App — (8)

(ii) Evaluate f(z) $= \frac{7z - 2}{z(z+1)(z-2)}$ in Laurent's series valid in the region $1 < |z + 1| < 3$ — CO4-App — (8)

Or

(b) Using Contour integration Prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a + b}$ $a > b > 0$ — CO4-App — (16)

20. (a) (i) Solve : $(D^2 - 3DD' + 2D'^2)Z = e^{3x-2y} + Sin(3x + 2y)$ — CO5-App — (8)

(ii) Solve : $x(y - z)p + y(z - x)q = z(x - y)$ — CO5- App — (8)

Or

(b) A tightly String with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If its set vibrating giving each point at velocity $\lambda(lx - x^2)$. Determine the displacement function y(x,t). — CO5- App — (16)