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**Question Paper Code: U2M10**

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Second Semester

Computer Science and Design

21UMA210- Differential equations, Fourier series & Transforms

(Regulations 2021)

(Common to Artificial Intelligence and Data Science)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- The auxiliary equation of the equation  $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$  is \_\_\_\_\_. CO1-App  
(a)  $m^2 - 4m + 5 = 0$       (b)  $m^2 + 3m - 2 = 0$       (c)  $m^2 + 3m + 2 = 0$       (d)  $2m^2 + 5m - 7 = 0$
- $\frac{1}{D^2}(\cos x) =$  \_\_\_\_\_. CO6-R  
(a)  $\sin x$       (b)  $-\cos x$       (c)  $\cos x$       (d)  $\tan x$
- $L(\sin h at) =$  \_\_\_\_\_. CO2-App  
(a)  $\frac{s}{s^2 - a^2}$       (b)  $\frac{a}{s^2 - a^2}$       (c)  $\frac{s}{s^2 + a^2}$       (d)  $\frac{a}{s^2 + a^2}$
- $L(te^{-3t})$  CO2-App  
(a)  $-z \frac{d}{dz} F(Z)$       (b)  $z \frac{d}{dz} F(Z)$       (c)  $\frac{d}{dz} F(Z)$       (d)  $z^n \frac{d}{dz} F(Z)$
- $Z[n f(n)] =$  \_\_\_\_\_. CO3- App  
(a)  $-z \frac{d}{dz} F(Z)$       (b)  $z \frac{d}{dz} F(Z)$       (c)  $\frac{d}{dz} F(Z)$       (d)  $z^n \frac{d}{dz} F(Z)$
- The Z transform of  $n2^n$  is \_\_\_\_\_. CO3- App  
(a)  $\frac{2z}{(z-2)^2}$       (b)  $\frac{z}{(z-2)^2}$       (c)  $\frac{2z}{(z+2)^2}$       (d)  $\frac{z}{(z+2)^2}$

7. The root mean square value of  $f(x) = x^2$  in  $(0, 1)$  is -----.
- (a) 1                                      (b)  $\frac{1}{2}$                                       (c)  $1/\sqrt{3}$                                       (d) 2                                      CO4-App
8. The constant term in the Fourier expansion for  $f(x) = k$ ,  $(0, 2\pi)$  is
- (a)  $2k$                                       (b)  $3k$                                       (c)  $k$                                       (d) 0                                      CO4-App
9. Convolution theorem on Fourier Transform is  $F[f(x)*g(x)] =$
- \_\_\_\_\_
- (a)  $F(s).G(s)$                                       (b)  $f(s).g(s)$                                       (c)  $F(s)*G(s)$                                       (d)  $f(s)*g(s)$                                       CO6-U
10. If  $f(x)$  is an even function then  $\int_{-a}^a f(x)dx =$  \_\_\_\_\_
- (a) 0                                      (b)  $\frac{1}{2} \int_0^a f(x)dx$                                       (c)  $2 \int_0^a f(x)dx$                                       (d)  $\int_0^a f(x)dx$                                       CO6-U

PART – B (5 x 2= 10Marks)

11. Find Particular integral for
- $(D^2 - 2D + 1)y = \cosh x$                                       CO1-App
12. Evaluate  $L[\cos at]$                                       CO2-App
13. Evaluate  $z\left(\frac{1}{n+1}\right)$                                       CO3-App
14. Give the expression for the Fourier series coefficient  $b_n$  for the function  $f(x) = x^2$  defined in  $-2 \leq x \leq 2$ .                                      CO4-App
15. Find the Fourier Sine Transform of  $f(x) = e^{-x}, x > 0$                                       CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve  $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$                                       CO1-App                                      (8)
- (ii) Solve  $(D^2 - D - 6)y = 3e^{4x} + 5$                                       CO1- App                                      (8)
- Or
- (b) (i) Solve  $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$                                       CO1- App                                      (8)
- (ii) Solve  $(3z - 4y)p + (4x - 2z)q = 2y - 3x$                                       CO1- App                                      (8)
17. (a) Find the Laplace transform of  $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$  and  $f(t + 2a) = f(t)$                                       CO2-App                                      (16)

Or

- (b) Solve by the convolution theorem CO2 -App (16)

$$L^{-1} \left[ \frac{s}{(s^2 + a^2)^2} \right]$$

18. (a) Using convolution theorem find the inverse Z-transform CO3-App (16)

$$Z^{-1} \left( \frac{8z^2}{(2z-1)(4z-1)} \right)$$

Or

- (b) (i) Evaluate  $Z[a^n \cos n\theta]$  and  $Z[a^n \sin n\theta]$  CO3-App (8)

- (ii) Evaluate CO3-App (8)

$$Z^{-1} \left[ \frac{z}{z^2 + 4z + 3} \right]$$

19. (a) Find the Fourier series of  $f(x) = x^2$  in  $(-\pi, \pi)$  of periodicity  $2\pi$ . CO4-App (16)

Or

- (b) (i) The table of values of the function  $y = f(x)$  is given below: CO4-App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent  $f(x)$  in terms of  $x$ .

- (ii) Find the half range sine series for  $f(x) = x^2$  in  $(0, l)$  CO4-App (8)

20. (a) Find the Fourier transform of  $f(x) = \begin{cases} 1 - |x|, & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$  and CO5-App (16)

hence deduce that i)  $\int_0^{\infty} \left( \frac{\sin t}{t} \right)^2 dt = \frac{\pi}{2}$     ii)  $\int_0^{\infty} \left( \frac{\sin t}{t} \right)^4 dt = \frac{\pi}{3}$

Or

- (b) (i) Find the Fourier sine & cosine transform of  $e^{-ax}$  CO5-App (8)

- (ii) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + 25)^2}$  CO5-App (8)

