A   **Reg. No. :** [   ]

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Question Paper Code: U1M02

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B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

First Semester

Civil Engineering

21UMA102- MATRIX AND CALCULUS

(Common to ALL branches)

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The sum and product of the Eigen values of $A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are _____   CO1- App

   (a) 4, -4   (b) 0, -4   (c) 4, 2   (d) 5, 3

2. If 0,3,4 are the Eigen values of a matrix A then $|A| = $ ___   CO6- U

   (a) 0   (b) 1   (c) 2   (d) 3

3. If $y = \cos(ax+b)$ then nth derivative y_n = _____   CO2- App

   (a) $a^n \cos\left(ax - b - \frac{n\pi}{2}\right)$ (b) $a^n \sin(ax + b)$ (c) $a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$ (d) $\cos(ax + b)$

4. If $E = 90$, $R = 12$, $L = 3$, then Kirchhoff's Law is _____   CO2- App

   (a) $\frac{di}{dt} + 4i = 30$ (b) $3\frac{di}{dt} + 90i = 12$ (c) $4\frac{di}{dt} + i = 30$ (d) $30\frac{di}{dt} + 4i = 1$

5. If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} = $ _____   CO3 - App

   (a) θ (b) r (c) $\frac{1}{r}$ (d) $\frac{1}{\theta}$

6. If $u = \cos^{-1}(\frac{x + y}{\sqrt{x} + \sqrt{y}})$, then cos u is a homogeneous function of degree _____   CO3- App

    (a) $\frac{1}{2}$ (b) 1   (c) -1   (d) 0

7. $\int_{0}^{\frac{\pi}{2}} \sin^6 x \, dx$ is _____   CO4- App

    (a) $\frac{\pi}{85}$ (b) $\frac{3}{256}$ (c) $\frac{\pi}{32}$ (d) $\frac{5\pi}{32}$

8. $\int_{0}^{\infty} e^{-x} x^4 dx =$ CO4 - App

    (a) 4   (b)4!   (c)5   (d) 5!

9. The region of integration of the integral $\int_{0}^{1} \int_{0}^{x} f(x, y) dx dy$ is   CO5 - App

    (a) square   (b) rectangle   (c) triangle   (d) circle

10. $\int_{0}^{1}\int_{0}^{2}\int_{0}^{3} dx dy dz$ is equal to   CO5 - App

    (a) 2   (b) 3   (c) 4   (d) 6

<div align="center">PART – B (5 x 2= 10Marks)</div>

11. Find the Eigen values of A^{-1} and $A^2 + 3I$ for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ CO1-App

12. Evaluate $\underset{x \to 0}{lt} \dfrac{6^x - 3^x}{x}$ CO2- App

13. If $u = \dfrac{x}{y} + \dfrac{y}{z} + \dfrac{z}{x}$ find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$ CO3- App

14. Compute $\Gamma\left(\dfrac{7}{2}\right)$ CO4- App

15. Evaluate $\int_{0}^{1}\int_{0}^{3} (x + y) dy dx$ CO5- App

16. (a) (i) Find the Eigen values and Eigen Vectors of $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ CO1- App   (8)

(ii) Using Cayley-Hamilton theorem find   CO1-App   (8)

A^4 for $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

Or

(b) Reduce the Q.F $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to a canonical form by an orthogonal transformation and hence find rank, signature, index and nature   CO1- App   (16)

17. (a) (i) A pot of boiling water 100°C is removed from the fire and allowed to cool at 30°C room temperature. Two minutes later, the temperature of the water in the pot is 90°C. What will be the temperature of the water after 5 minutes?   CO2- Ana   (8)

(ii) Expand $e^{\sin x}$ by Maclaurin's series up to the term containing x^4 CO2- App   (8)

Or

(b) (i) If $y = e^{ax} \cos bx$, prove that $\dfrac{d^2 y}{dx^2} - 2a\dfrac{dy}{dx} + (a^2 + b^2)y = 0$ CO2- App   (8)

(ii) Find the n$^{\text{th}}$ derivative of $\dfrac{x^2 + x - 1}{x^3 + x^2 - 6x}$ CO2- App   (8)

18. (a) (i) A rectangular box open at the top is to have a volume of 32cc, calculate the dimensions of the box that requires least material for its constructions   CO3- App   (10)

(ii) If $u = \dfrac{yz}{x}$, $v = \dfrac{xz}{y}$, $w = \dfrac{xy}{z}$ then show that $\dfrac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)} = 4$ CO3- App   (6)

Or

(b) (i) Using Taylor's series expand $e^x \sin y$ near the point $\left(-1, \dfrac{\pi}{4}\right)$ up to third degree terms.   CO3- App   (8)

(ii) Obtain the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$ CO3- App   (8)

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19. (a) (i) Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ CO4- App (10)

   (ii) Compute $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{\sin x}}{\sqrt{(\cos x)} + \sqrt{(\sin x)}}\,dx$ CO4- App (6)

                               Or

   (b) (i) Determine the reduction formula for $\displaystyle\int \cos^n x\,dx$ CO4- App (10)

   (ii) Prove that $\beta\left(m, \dfrac{1}{2}\right) = 2^{2m-1}\,\beta(m,\,m)$ CO4- App (6)

20. (a) Using the Triple integration, compute the volume of the   CO5- App (16)
   Ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

                               Or

   (b) (i) Show that the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$ CO5- App (8)
   is $\dfrac{16}{3}a^2$

   (ii) Change the order of integration and hence evaluate   CO5- App (8)
   $\displaystyle\int_0^a \int_x^a \left(x^2 + y^2\right)dy\,dx$

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