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Question Paper Code: U2M06

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Second Semester

Civil Engineering

21UMA206- Differential Equations, Complex Analysis & Transform Techniques

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- $\frac{1}{(D-m)^2} e^{mx} = \underline{\hspace{2cm}}$ CO1-App
(a) $x e^{mx}$ (b) $x^2 e^{mx}$ (c) $\frac{x^2}{2} e^{mx}$ (d) $\frac{x^2}{m} e^{mx}$
- The complementary function of $(4D^2 - 3D - 1)y = 2 \sin 2x$ is $\underline{\hspace{2cm}}$ CO1-App
(a) $Ae^x + Be^{\frac{x}{4}}$ (b) $Ae^{-x} + Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^x + Be^{4x}$
- If $\phi = x^2 + y^2 - z - 10$ then $|\nabla \phi|$ at $(1, 1, 1)$ is $\underline{\hspace{2cm}}$ CO2-App
(a) $2(\bar{i} + \bar{j} + \bar{k})$ (b) $2\bar{i} + 2\bar{j} - \bar{k}$ (c) 3 (d) 9
- If ϕ is a vector point function then $\text{Curl}(\text{grad}\phi) = \underline{\hspace{2cm}}$ CO2-App
(a) 1 (b) 0 (c) 2 (d) None of the above
- The PDE obtained from $z = (x+a)(y+b)$ is $\underline{\hspace{2cm}}$. CO3- App
(a) $3z = px + qy$ (b) $py - qx = 0$ (c) $z = pq$ (d) $px+qy = 0$
- The PDE of all planes having equal intercepts on the X axis and Y axis is $\underline{\hspace{2cm}}$ CO3- App
(a) $p = q$ (b) $p + q = 0$ (c) $pq = 1$ (d) $p(q + 1) = q$
- Simple pole is a pole of order $\underline{\hspace{2cm}}$ CO6-U
(a) 1 (b) 2 (c) 3 (d) 4

8. The poles of $z \cot z$ is _____ CO6-U
 (a) 0 (b) $\pm n\pi$ (c) 1 (d) π
9. $L(\sinh at) =$ _____ CO6-R
 (a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$
10. $L[t f(t)] =$ _____ CO6-R
 (a) $F'(s)$ (b) $-F'(s)$ (c) $F(s)$ (d) $-F(s)$

PART – B (5 x 2= 10Marks)

11. Compute the particular integral for $(D^2 - 2D + 1)y = \cosh x$. CO1-App
12. Compute the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at $(1, 1, 1)$. CO2-App
13. Compute the complete integral of $p - q = k$ CO3-App
14. State Cauchy's residue theorem. CO6-R
15. Verify initial value theorem for the function $1 + e^{-2t}$. CO6-U

PART – C (5 x 16= 80Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + 4)y = \sec 2x$. CO1-App (8)
 (ii) A colony of bacteria is growing exponentially. At time $t=0$ it has 10 bacteria in it, and at time $t=4$ it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)
- Or
- (b) (i) Solve: $(x^2 D^2 + xD + 1)y = x \sin(\log x)$ CO1- App (8)
 (ii) Solve: $(D^2 - 4D + 3)y = \sin 3x + e^{2x}$ CO1- App (8)
17. (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped $x = 0, x = a, y = 0, y = b, z = 0, z = c$. CO2-App (16)
- Or
- (b) (i) Using Green's theorem, Evaluate $\int_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is the boundary of the region defined by $X = 0, Y = 0, X + Y = 1$ in the XY plane. CO2 -App (8)
 (ii) Prove that $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ is irrotational vector and compute the Scalar potential such that $\vec{F} = \nabla\phi$. CO2 -App (8)

18. (a) (i) Solve: $(mz - ny)p + (nx - lz)q = ly - mx$ CO3-App (8)
(ii) Solve $(D^2 - DD' - 3D'^2)z = \sin(x + y) + e^{6x+y}$ CO3-App (8)
Or
- (b) (i) Solve $p^2 + q^2 = x^2 + y^2$ CO3-App (8)
(ii) Form a PDE by eliminating arbitrary functions from CO3-App (8)
 $z = px + qy + p^2 - q^2$
19. (a) CO4-App (8)
(i) Evaluate $f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ by using Cauchy's
Integral formula where C is $|z| = 3$
- (ii) Expand $\frac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the region CO4-App (8)
 $2 < |z| < 3$
Or
- (b) Using Contour integration, to prove CO4-App (16)

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a+b} \quad a > b > 0$$
20. (a) (i) Solve the differential equation $\frac{d^2y}{dt^2} + y = \sin 2t$; $y(0) = 0$; $y'(0) = 0$ by using Laplace transform method. CO5-App (8)
(ii) Compute the Laplace Transforms of $\frac{\cos at - \cos bt}{t}$ CO5-App (8)
Or
- (b) (i) Using the periodic function, Compute the Laplace transform of CO5-App (8)

$$f(t) = f(t) = \begin{cases} k, & 0 \leq t \leq a \\ -k, & a \leq t \leq 2a \end{cases}$$
- (ii) Using Convolution Theorem, Compute $L^{-1} \left[\frac{1}{(s^2 + a^2)(s^2 + b^2)} \right]$ CO5-App (8)

