

**A****Reg. No. :**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

**Question Paper Code: U4M23**

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Civil Engineering

21UMA423 - Numerical Methods

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

|    |  |                      |             |                   |
|----|--|----------------------|-------------|-------------------|
| 1. | Order of convergence of Newton's method is _____   |                      |             | CO1-U             |
|    | (a) 4  | (b) 1                | (c) 2       | (d) 3             |
| 2. | When Gauss Elimination method is used to solve $AX=B$ , A is transferred in a _____ matrix         |                      |             | CO1- U            |
|    | (a) lower triangular   | (b) upper triangular | (c) square  | (d) zero          |
| 3. | In Cubic Spline, $M_0=M_n=$ _____  |                      |             | CO2- U            |
|    | (a) 1  | (b)n                 | (c)3        | (d) 0             |
| 4. | In Cubic Spline, $M_0=M_n=$ _____  |                      |             | CO2- U            |
|    | (a) 1  | (b)n                 | (c)3        | (d) 0             |
| 5. | Trapezoidal rule is so called, because it approximates the integral by the sum of _____ trapezoids |                      |             | CO3- U            |
|    | (a) n  | (b)n+1               | (c)n-1      | (d) 2n            |
| 6. | Gaussian three point quadrature formula is exact for polynomials up to degree _____                |                      |             | CO3- U            |
|    | (a) 1  | (b)2                 | (c)3        | (d) 5             |
| 7. | Predictor-Corrector methods are _____ starting methods   |                      |             | CO4- U            |
|    | (a) self   | (b)not self          | (c)identity | (d) None of these |
|    |  |                      |             |                   |

|                            |   |  |               |                   |   |   |    |   |   |    |     |  |
|----------------------------|---|--|---------------|-------------------|---|---|----|---|---|----|-----|--|
| 8.                         | _____ prior values are required to predict the next value in Milne's method   |  |               | CO4- U            |   |   |    |   |   |    |     |  |
|                            | (a) 1   | (b)2   | (c)3          | (d) 4             |   |   |    |   |   |    |     |  |
| 9.                         | PDE of second order, if $B^2-4AC= 0$ then   |  |               | CO6- U            |   |   |    |   |   |    |     |  |
|                            | (a) parabolic   | (b)elliptic  | (c)hyperbolic | (d) None of these |   |   |    |   |   |    |     |  |
| 10.                        | $u_{xx}+u_{yy}=0$ is a _____ equation   |  |               | CO5- U            |   |   |    |   |   |    |     |  |
|                            | (a) Laplace   | (b)Poisson   | (c)heat       | (d) wave          |   |   |    |   |   |    |     |  |
| PART – B (5 x 2= 10Marks)  |   |  |               |                   |   |   |    |   |   |    |     |  |
| 11.                        | Compare Gauss Elimination and Gauss Jordan Methods  |  |               | CO1- U            |   |   |    |   |   |    |     |  |
| 12.                        | Form the divided difference table for the following data  |  |               | CO2- App          |   |   |    |   |   |    |     |  |
|                            | <table border="1"> <tr> <td>x</td> <td>2</td> <td>5</td> <td>10</td> </tr> <tr> <td>y</td> <td>5</td> <td>29</td> <td>109</td> </tr> </table> |  |               | x                 | 2 | 5 | 10 | y | 5 | 29 | 109 |  |
| x                          | 2   | 5  | 10            |                   |   |   |    |   |   |    |     |  |
| y                          | 5   | 29   | 109           |                   |   |   |    |   |   |    |     |  |
| 13.                        | Using two –point Gaussian quadrature formula find $\int_{-1}^1 \frac{1}{1+x^2} dx$  |  |               | CO3- App          |   |   |    |   |   |    |     |  |
| 14.                        | Using Euler's method find $y(0.1)$ given $\frac{dy}{dx} = 1+ y^2$ , $y(0) = 0$  |  |               | CO4- App          |   |   |    |   |   |    |     |  |
| 15.                        | Classify $u_{xx} - 2u_{xy} + u_{yy} = 0$  |  |               | CO6- U            |   |   |    |   |   |    |     |  |
| PART – C (5 x 16= 80Marks) |   |  |               |                   |   |   |    |   |   |    |     |  |
| 16.                        | (a)   | (i) Solve the equation $e^x - 3x = 0$ by iteration method  |               | CO1-App (8)       |   |   |    |   |   |    |     |  |
|                            |   | (ii) Solve $x + 2y + z = 8$ , $2x + 3y + 4z = 20$ , $4x + y + 2z = 12$ using Gauss Elimination method  |               | CO1-App (8)       |   |   |    |   |   |    |     |  |
|                            |   | Or   |               |                   |   |   |    |   |   |    |     |  |
|                            | (b)   | (i) Using Power method find numerically largest Eigen value and the corresponding Eigen vector of the matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ |               | CO1 -App (8)      |   |   |    |   |   |    |     |  |
|                            |   | (ii) Solve $27x + 6y - z = 85$ , $6x + 15y + 2z = 72$ , $x + y + 54z = 110$  |               | CO1-App (8)       |   |   |    |   |   |    |     |  |

|     |     |   |    |     |   |   |    |    |    |    |          |     |          |     |
|-----|-----|---|----|-----|---|---|----|----|----|----|----------|-----|----------|-----|
|     |     | using Gauss Seidel method   |    |     |   |   |    |    |    |    |          |     |          |     |
|     |     |   |    |     |   |   |    |    |    |    |          |     |          |     |
| 17. | (a) | Using Lagrange's interpolation formula calculate $f(10)$ for the following data<br><br><table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X</td> <td>5</td> <td>6</td> <td>9</td> <td>11</td> </tr> <tr> <td>Y</td> <td>12</td> <td>13</td> <td>14</td> <td>16</td> </tr> </table>              | X  | 5   | 6 | 9 | 11 | Y  | 12 | 13 | 14       | 16  | CO2-App  | (8) |
| X   | 5   | 6   | 9  | 11  |   |   |    |    |    |    |          |     |          |     |
| Y   | 12  | 13  | 14 | 16  |   |   |    |    |    |    |          |     |          |     |
|     |     | (ii) Using Newton's divided difference formula calculate $f(3)$ satisfying the following data:<br><br><table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>5</td> </tr> <tr> <td>y</td> <td>2</td> <td>3</td> <td>12</td> <td>147</td> </tr> </table> | x  | 0   | 1 | 2 | 5  | y  | 2  | 3  | 12       | 147 | CO2- App | (8) |
| x   | 0   | 1   | 2  | 5   |   |   |    |    |    |    |          |     |          |     |
| y   | 2   | 3   | 12 | 147 |   |   |    |    |    |    |          |     |          |     |
|     |     | Or  |    |     |   |   |    |    |    |    |          |     |          |     |
|     | (b) | (i) Using Newton's backward interpolation formula calculate $f(4)$ from the following data :<br><br><table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>y</td> <td>1</td> <td>2</td> <td>1</td> <td>10</td> </tr> </table>     | x  | 0   | 1 | 2 | 3  | y  | 1  | 2  | 1        | 10  | CO2- App | (8) |
| x   | 0   | 1   | 2  | 3   |   |   |    |    |    |    |          |     |          |     |
| y   | 1   | 2   | 1  | 10  |   |   |    |    |    |    |          |     |          |     |
|     |     | (ii) Using cubic spline function calculate $f(1.5)$ for the following data<br><br><table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>y</td> <td>-8</td> <td>-1</td> <td>18</td> </tr> </table>   | x  | 1   | 2 | 3 | y  | -8 | -1 | 18 | CO2- App | (8) |          |     |
| x   | 1   | 2   | 3  |     |   |   |    |    |    |    |          |     |          |     |
| y   | -8  | -1  | 18 |     |   |   |    |    |    |    |          |     |          |     |
|     |     |   |    |     |   |   |    |    |    |    |          |     |          |     |
| 18. | (a) | (i) Compute the first and second derivatives of $y$ at $x = 1$ from<br><br><table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>y</td> <td>1</td> <td>8</td> <td>27</td> <td>64</td> </tr> </table>                             | x  | 1   | 2 | 3 | 4  | y  | 1  | 8  | 27       | 64  | CO3- App | (8) |
| x   | 1   | 2   | 3  | 4   |   |   |    |    |    |    |          |     |          |     |
| y   | 1   | 8   | 27 | 64  |   |   |    |    |    |    |          |     |          |     |

|     |     |  |          |     |
|-----|-----|--|----------|-----|
|     |     | (ii) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ with 6 equal intervals by<br><br>(a) Trapezoidal rule<br><br>(b) Simpson's $\frac{1}{3}$ rule.   | CO3- App | (8) |
|     |     | Or   |          |     |
|     | (b) | (i) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using Romberg's method correct to 4 decimal places.   | CO3- App | (8) |
|     |     | (ii) Evaluate $\int_0^1 \int_0^1 e^{(x+y)} dx dy$ using Trapezoidal rule by taking $h=k=0.5$   | CO3- App | (8) |
|     |     |  |          |     |
|     |     |  |          |     |
| 19. | (a) | (i) Using Taylor's series method find $y(1.1)$ given $y' = x + y$ with $y(1) = 0$  | CO4- App | (8) |
|     |     | (ii) Given $\frac{dy}{dx} = 1 + y^2$ , $y(0) = 0$ , $y(0.2) = 0.2027$ , $y(0.4) = 0.4228$ , $y(0.6) = 0.684$ evaluate $y(0.8)$ by Milne's Method   | CO4- App | (8) |
|     |     | Or   |          |     |
|     | (b) | (i) Using R-K method of fourth order, find $y(0.1)$ for the initial value problem $\frac{dy}{dx} = x + y^2$ with $y(0) = 1$  | CO4- App | (8) |
|     |     | (ii) Using Adam's Bash forth Predictor-Corrector method, find $y(4.4)$ given that $5xy' + y^2 = 2$ , $y(4) = 1$ , $y(4.1) = 1.0049$ , $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$                                | CO4- App | (8) |
|     |     |  |          |     |
| 20. | (a) | (i) Solve $\frac{\partial^2 u}{\partial x^2} = 2 \frac{\partial u}{\partial t}$ , $u(0,t) = 0$ , $u(4,t) = 0$ , $u(x,0) = x(4 - x)$ .<br>Take $h = 1$ and find the values of $u$ up to $t = 5$ using Bender- | CO5- App | (8) |

|  |     |   |          |      |
|--|-----|---|----------|------|
|  |     | Schmidt's difference equation   |          |      |
|  |     | (ii) Using Crank-Nicholson's difference equation to solve<br>$\frac{\partial^2 u}{\partial x^2} = 16 \frac{\partial u}{\partial t}$<br>$u(0,t) = 0$ , $u(1,t) = 100t$ , $u(x,0) = 0$ compute $u$ for one time step function with $h=0.25$ | CO5- App | (8)  |
|  |     | Or  |          |      |
|  | (b) | Solve $\nabla^2 \mathbf{u} = -10(\mathbf{x}^2 + \mathbf{y}^2 + 10)$ over the square mesh with sides $\mathbf{x} = 0, \mathbf{x} = 3, \mathbf{y} = 0, \mathbf{y} = 3$ with $u=0$ on the boundary and mesh length 1 unit.                   | CO5- App | (16) |