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Question Paper Code: U3026

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Third Semester

Agricultural Engineering

21UMA326- TRANSFORM TECHNIQUES AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Biomedical and Biotechnology Engineering)

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer All Questions

PART A - (10 x 1 = 10 Marks)

- The term $(a_1 \cos x + b_1 \sin x)$ in Fourier series is called _____ CO6 -U
(a) First harmonic (b) Second harmonic
(c) Third harmonic (d) Fourier Coefficients
- If a function $f(x)$ is even, its Fourier expansion contains only _____ terms. CO6- U
(a) Sine (b) Cosine (c) tan (d) None of these
- If $F[f(x)] = f(s)$ then the function is said to be _____ CO6 -U
(a) Odd (b) Even (c) Self Reciprocal (d) Periodic
- If $f(x)$ is an odd function then $\int_{-a}^a f(x) dx =$ _____ CO6- U
(a) 0 (b) $\frac{1}{2} \int_0^a f(x) dx$ (c) $2 \int_0^a f(x) dx$ (d) $\int_0^a f(x) dx$
- The difference equation of $z (e^{an})$ CO3- App
(a) $\frac{z}{z + e^a}$ (b) $\frac{z^2}{z + e^a}$ (c) $\frac{z}{z - e^a}$ (d) $\frac{z^2}{z - e^a}$
- $Z [n f(n)] =$ _____ CO3 -App
(a) $-z \frac{d}{dz} F(Z)$ (b) $z \frac{d}{dz} F(Z)$ (c) $\frac{d}{dz} F(Z)$ (d) $z^n \frac{d}{dz} F(Z)$

7. The general solution of $2r + 5s - 3t = 0$ is ____ CO4- App
 (a) $f_1(y + 3x) + f_2(2y - x)$ (b) $f_1(3y + x) + f_2(y + 2x)$
 (c) $f_1(y - 3x) + f_2(2y - x)$ (d) none of these
8. The PDE obtained from $z = (x+a)(y+b)$ is ____ CO4- App
 (a) $3z = px + qy$ (b) $py - qx = 0$ (c) $z = pq$ (d) $px + qy = 0$
9. In a one dimensional wave equation, $c^2 =$ ____ CO6- U
 (a) T^2/m^2 (b) T/m (c) T/m^2 (d) T^2/m
10. Classify the equation $u_{xx} + u_{yy} = 0$ is ____ CO5- U
 (a) parabolic (b) hyperbolic (c) elliptic (d) cyclic

PART – B (5 x 2 = 10 Marks)

11. Find the root mean square value of the function $f(x) = x$ in $(0, 1)$ CO1- App
 12. Write Fourier sine Transform pair CO6- U
 13. Evaluate $z(n)$ CO3 -App
 14. Find the Particular Integral of $(D^2 - 2DD' + 2D'^2)z = \sin(x - y)$ CO4 -App
 15. Write the constant term in the one dimensional heat equation CO6- U

PART – C (5 x 16 = 80 Marks)

16. (a) The table of values of the function $y = f(x)$ is given below: CO1- App (16)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Find a Fourier series upto the third harmonic to represent $f(x)$ in terms of x .

Or

- (b) (i) Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π . CO1 -App (8)
 Hence deduce that the value of the sum.
 (ii) Find the Half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. CO1 -App (8)

Deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$

17. (a) Compute the Fourier Transform of $f(x) = \begin{cases} a - |x| & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$ and CO2- App (16)
- hence evaluate (i) $\int_0^{\infty} \left(\frac{\sin x}{x}\right)^4 dx$ (ii) $\int_0^{\infty} \left(\frac{\sin x}{x}\right)^2 dx$
- Or
- (b) Evaluate (i) $\int_0^{\infty} \frac{dx}{(x^2+16)^2}$ (ii) $\int_0^{\infty} \frac{dx}{(x^2+49)^2}$ using Fourier transform CO2- App (16)
18. (a) (i) Solve the difference equation $y_{n+2} - 6y_{n+1} + 8y_n = 5^n$ given CO3- App (8)
that $y_0 = 0, y_1 = 0$
- (ii) Using Convolution theorem find CO3- App (8)
- $$z^{-1} \left[\frac{10z^2}{(5z-2)(2z+1)} \right]$$
- Or
- (b) (i) Solve the difference equation $y_{n+2} + 3y_{n+1} - 10y_n = 3^n$ given CO3- App (8)
that $y_0 = 0, y_1 = 0$
- (ii) Using Convolution theorem find CO3- App (8)
- $$z^{-1} \left[\frac{12z^2}{(3z-1)(4z-1)} \right]$$
19. (a) (i) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ CO4 - App (8)
- (ii) Solve $(D^2 - 4DD' + 4D'^2)z = e^{2x+y} + \sin(3x + 4y)$ CO4 - App (8)
- Or
- (b) (i) Solve $z = px + qy + p^2q^2$ CO4 -App (8)
- (ii) Form the partial differential equation by eliminating the arbitrary function f and g in $z = x^2 f(y) + y^2 g(x)$ CO4 -App (8)
20. (a) A String is stretched and fastened to two points l apart .Motion is started by displacing the string into the form $y=K x(l-x)$ from which it is released at $t=0$.Find the displacement of any point at a distance 'x' at any time 't'. CO5 -App (16)
- Or

- (b) A bar of 10cm long with insulated sides has its ends A and B kept at 20°C and 40°C respectively. Until steady state condition prevails. The temperature at A is then suddenly raised to 50°C and at the same instant B is lowered to 10°C and maintained thereafter. Find the subsequent temperature distribution in the bar. CO5- App (16)