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**Reg. No. :**

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**Question Paper Code: U2M11**

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Second Semester

Artificial Intelligence and Machine Learning

21UMA211-FOURIER SERIES, PARTIAL DIFFERENTIAL EQUATIONS AND

COMPLEX ANALYSIS

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The term  $(a_1 \cos x + b_1 \sin x)$  in Fourier series is called ----- CO6- U  
(a) First harmonic (b) Second harmonic (c) Third harmonic (d) Fourier coefficients
2. The root mean square value of  $f(x)$  in  $(0, l)$  is ----- CO1- App  
(a) 1 (b)  $l/2$  (c)  $l/\sqrt{3}$  (d)  $2l$
3. The PDE obtained from  $z = (x+a)(y+b)$  is \_\_\_\_\_. CO2-App  
(a)  $3z = px + qy$  (b)  $py - qx = 0$  (c)  $z = pq$  (d)  $px+qy = 0$
4. The general solution of  $(D^2 - 8DD' + 12D'^2) z = 0$  is \_\_\_\_\_ CO2-App  
(a)  $f_1(y + 2x) + f_2(y + 6x)$  (b)  $f_1(y + 2x) + f_2(y - 6x)$   
(c)  $z = xf(y + 3x/2) - g(y + 3x/2)$  (d)  $z = xf(y - 3x/2)$
5. Classify the equation  $u_{xx} + u_{yy} = 0$  is \_\_\_\_\_ CO6- U  
(a) parabolic (b) hyperbolic (c) elliptic (d) cyclic
6. An insulated rod of length 60 cm has its ends at A and B kept at  $20^\circ\text{C}$  and  $80^\circ\text{C}$  respectively, then its steady state solution is CO3-App  
(a)  $x - 20$  (b)  $4x + 20$  (c)  $x + 20$  (d)  $x + 60$
7. The critical point of the transformation  $w = z + \frac{1}{z}$  are \_\_\_\_\_ CO4- App  
(a)  $\pm 1$  (b)  $\pm 2$  (c)  $\pm 1$  (d)  $\pm 2$

8. Find the fixed points of  $f(z) = \frac{1}{z - 2i}$  CO4-App
- (a) i (b) 2i (c) 3i (d) 0
9. The residue of  $f(z) = \frac{4}{z^3(z - 2)}$  at its simple pole is \_\_\_\_\_ CO5-App
- (a)  $\frac{4}{7}$  (b)  $\frac{3}{4}$  (c)  $\frac{1}{7}$  (d)  $\frac{3}{4}$
10. The value of  $\int_c \frac{dz}{z+2}$ ,  $c: |z| = 1$  is \_\_\_\_\_ CO6-U
- (a)  $2\pi i$  (b) 0 (c)  $4\pi i$  (d) 0

PART – B (5 x 2= 10Marks)

11. State Dirichlet's conditions CO1-App
12. Solve the PDE  $pq = 8$ . CO2-App
13. Classify  $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8u_y - 16u = 0$  CO3-App
14. Find the fixed point of  $w = \frac{2z - 5}{z + 4}$  CO4-App
15. Expand  $\frac{1}{z(z - 1)}$  as a Laurent series about  $z=0$  in the annulus  $0 < |z| < 1$ . CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Express  $f(x) = \frac{1}{2}(\pi - x)$  as a Fourier series of period  $2\pi$  in the CO1- App (8)  
internal  $0 < x < 2\pi$ .

- (ii) The table of values of the function  $y = f(x)$  is given below: CO1-App (8)

x	0	$\pi/3$	$2\pi/3$	$\pi$	$4\pi/3$	$5\pi/3$	$2\pi$
y:	1.8	0.3	0.5	2.16	1.3	1.76	1.8

Find a Fourier series up to the third harmonic to represent  $f(x)$  in terms of  $x$ .

Or

- (b) (i) Express  $f(x) = x^2$  as a Fourier series of period  $2\pi$  in the CO1- App (8)  
interval  $0 < x < 2\pi$ .

(ii) The table of values of the function  $y = f(x)$  is given below: CO1-App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent  $f(x)$  in terms of  $x$

17. (a) (i) Solve  $(D^2 - DD^1 - 2D^{1^2})z = e^{3x+4y} + \cos(x+y)$  CO2 -App (8)

(ii) Solve  $x(y-z)p + y(z-x)q = z(x-y)$  CO2 -App (8)

Or

(b) (i) Solve  $Z = px + qy + \sqrt{pq}$  CO2 -App (8)

(ii) Form a PDE by eliminating arbitrary functions from CO2 -App (8)

$$\varphi(x^2 + y^2 + z^2, x + y + z) = 0..$$

18. (a) A bar of 10cm long with insulated sides has its ends A and B kept CO3-App (16)

at  $50^\circ\text{C}$  and  $100^\circ\text{C}$  respectively. Until steady state condition prevails. The temperature at A is then suddenly raised to  $90^\circ\text{C}$  and at the same instant B is lower to  $60^\circ\text{C}$  and maintained thereafter. Find the subsequent temperature distribution in the bar.

Or

(b) A String is stretched and fastened to two points 1 apart. Motion is CO3-App (16)

started by displacing the string into the form  $y=K(lx-x^2)$  from which it is released at  $t=0$ . Find the displacement of any point at a distance 'x' at any time 't'

19. (a) (i) Using Milne Thomson method, find the Analytic function CO4-App (8)

given that 
$$u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$$

(ii) Find the bilinear transformation from  $-1,0,1$  to  $0,i,3i$  CO4-App (8)

Or

(b) (i) Find the image of  $|z-1|=1$  under the transformation CO4-App (8)

$$w = \frac{1}{z}$$

(ii) If  $f(z)$  is analytic whose real part is constant must itself be a constant CO4-App (8)

20. (a) (i) Evaluate

CO5-App (8)

$$f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z+2)} dz \text{ by using Cauchy's Integral formula}$$

where C is  $|z| = 3$

(ii) (ii) Evaluate  $f(z) = \frac{1}{(z+1)(z+3)}$  in Laurent series valid for the region

CO5-App (8)

$$1 < |z| < 3.$$

Or

(b) Using Contour integration, to prove

CO5-App (16)

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a+b} \quad a > b > 0$$