A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U5M01

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Fifth Semester

Artificial Intelligence & Data Science

21UMA501 - LINEAR ALGEBRA

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. In a vector space V, for every $x, y \in$ V then the property $x + y = y + x$ is known as _________ CO6-U

   (a) commutative (b) associative (c) identity (d) inverse

2. The trivial subspaces of a vector space V are _________ CO6-U

   (a) {0} (b) V (c) W (d) {0}and V

3. If T: V→W be linear transformation then T(0) = _________ CO6- U

   (a) 0 (b) 1 (c) 2 (d) 3

4. In a linear transformation T: V→W the kernel of T is a subspace of _________ CO6- U

   (a) V (b) W (c) both V and W (d) none of these

5. $\langle x, x \rangle = 0$ if and only if _________ CO6- U

   (a) $x = 1$ (b) $x \neq 1$ (c) $x = 0$ (d) $x \neq 0$

6. In a vector space V , if $\langle x, y \rangle = \langle y, z \rangle$ then _________ CO6- U

   (a) $y = z$ (b) $y \neq z$ (c) $y = -z$ (d) none of these

7. The Hermitian Matrices containing some _________ numbers. CO6- U

   (a) Real (b) Imaginary (c) Natural (d) None of these

8. Every self - adjoint mapping is _________ CO6- U

(a) Linear (b) Co - factor (c) None Linear (d) none of these

9. A square matrix A is said to be ------------ if the determinant value of A is zero. CO6- U

(a) Singular (b) Non - Singular (c) Symmetric (d) none of these

10. Linear Programming deals with the ----------- of a function of decision variable. CO6- U

(a) Optimization (b) Formulation (c) Technique (d) none of these

PART – B (5 x 2= 10Marks)

11. Verify the commutative property for a vector space $R \times R$ over R under addition defined by $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ CO1-App

12. State any two properties of linear transformation CO6-U

13. Find the norm of $(2,1,-1)$ in $V_3(R)$ with standard inner product. CO3 -App

14. Define Cholesky decomposition CO6-U

15. Find the characteristic Polynomial of CO5- App

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

PART – C (5 x 16= 80Marks)

16.	(a)	(i) Verify the vectors (2,1,0) , (-3,-3,1) ,(-2,1,-1) in R^3 is a basis of R^3	CO1- App	(8)
		(ii) Prove that R X R is a Vector Space over under addition and Scalar multiplication defined by $(x_1,x_2) + (y_1,y_2) = (x_1+y_1, x_2+y_2)$ and $\alpha(x_1,x_2) = (\alpha x_1,\alpha x_2)$	CO1- App	(8)
		Or		
	(b)	(i) Find the dimension of the subspace spanned by the vectors (1,2,-3) ,(2,5,1) ,(-1,1,4) in $V_3(R)$	CO1 -App	(8)
		(ii) In R^3 determine whether (5,1,-5) is expressed as a linear combination of (1,-2,-3) and (-2,3,-4)	CO1 -App	(8)
17.	(a)	(i) If T: $R^2 \rightarrow R^2$ be linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ then find nullity(T) ,rank(T), Is T one-to-one? Is T onto? Also check the rank nullity theorem.	CO2- App	(8)
		(ii) Find the matrix of the linear transformation T: $R^2 \rightarrow R^2$ defined by T(a,b) =(2a-3b,a+b) for the standard basis of R^2	CO2- App	(8)
		Or		

	(b)	T: $V_1(R) \rightarrow V_3(R)$ is defined by $T(a_1) = (a_1, 2a_1^2, a_1^3)$ Verify whether T is a linear transformation	CO2- App	(16)
18.	(a)	(i) Show that the following function defines an inner product on $V_2(R)$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$ and $\langle x, y \rangle = 6x_1y_1 + 7x_2y_2$	CO3- App	(8)
		(ii) If $x = (2, 1+i, i)$ and $y = (2-i, 2, 1+2i)$ then verify Schwarz's inequality.	CO3- App	(8)
		Or		
	(b)	Apply Gram-Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1 = (1,0,1)$, $v_2 = (1,3,1)$ and $v_3 = (3,2,1)$	CO3- App	(16)
19.	(a)	Determine the Cholesky Decomposition for $A = \begin{pmatrix} 4 & 2i & -i \\ -2i & 10 & 1 \\ i & 1 & 9 \end{pmatrix}$	CO4- App	(16)
		Or		
	(b)	Determine the Cholesky Decomposition for $A = \begin{pmatrix} 16 & -3 & 5 & -8 \\ -3 & 16 & -5 & -8 \\ 5 & -5 & 24 & 0 \\ -8 & -8 & 0 & 21 \end{pmatrix}$	CO4- App	(16)
20.	(a)	Diagonalise the Matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ and hence find A^4.	CO5- App	(16)
		Or		
	(b)	Discuss by an Orthogonal reduction of $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$	CO5- App	(16)