A

Reg. No. : | | | | | | | | | | | | |

Question Paper Code: U2M07

B.E./B.Tech. DEGREE EXAMINATION, NOV 2023

Second Semester

Agriculture Engineering

21UMA207- CALCULUS COMPLEX ANALYSIS AND TRANSFORM TECHNIQUES

(Regulations 2021)

(Common to bio medical and biotechnology engineering branches)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The order and degree of $(y''')^2 + 2(y'')^3 + y = 0$ is _____ CO1-App

   (a) 3,2 (b)2,3 (c) 3,3 (d) 2,2

2. The solution of $(D^3 + D^2 - D - 1)y = 0$ is _____ CO6-U

   (a) $Ae^x + Bxe^x + Cx^2e^x$ (b) $(Ax + B)e^x + C\,e^{-x}$

   (c) $e^{-x} + (\cos 2x + i \sin 2x)$ (d) $(Ax + B)e^{-x} + C\,e^x$

3. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is _____ CO2-App

   (a) 8 (b) 4 (c) -3 (d) 0

4. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is _____ CO2-App

   (a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3 (d) 9

5. The critical point of the transformation $w = z + \frac{1}{z}$ are ____ CO6- U

   (a) ± 1 (b) ± 2 (c) $\pm i$ (d) $-i$

6. The mapping $w = z^2$ is not conformal at CO6- U

   (a) 0 (b) -1 (c) 1 (d) 2

7. Simple pole is a pole of order _____ CO6-U

   (a) 1 (a) 2 (a) 3 (a) 4

8. The poles of z cot z is _____ CO6-U

(a) 0 (b) ± nπ (c) 1 (d) π

9. L (sin h at) = _____ CO6-U

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. Sin t is a periodic function with period ____ CO6-U

(a) 2π (b) π (c) π / 2 (d) π / 3

PART – B (5 x 2= 10Marks)

11. Find the Particular Integral of $(D^2 + 4D + 4)y = \frac{e^{-2x}}{x^2}$ CO1-App

12. Find the Directional derivative of $\varphi = 4xz^2 + x^2yz$ at (1,-2,-1) in the direction $2\vec{i} + 3\vec{j} + 4\vec{k}$. CO2-App

13. Find the fixed point of $w = \frac{2z - 5}{z + 4}$ CO3-App

14. Evaluate $\int_c \frac{e^{-z}}{z+1} dz$ where C is $|z| = \frac{1}{2}$ using Cauchy integral formula CO4-App

15. Verify initial value theorem for the function $1+e^{-2t}$. CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + a^2)y =$ cosec ax. CO1-App (8)

(ii) Suppose a material decays at a rate proportional to the quantity of the material and there were 2200 grams 10 years ago. If there are 2000 grams now, what is the half-life? CO1- App (8)

Or

(b) (i) Solve: $(D^2 + 4D + 3)y = \sin x + x^2$ CO1- App (8)

(ii) Solve: $(D^2 - 4D + 3)y = \sin 3x + e^{2x}$ CO1- App (8)

17. (a) Verify Gauss Divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ where *S* is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0$ & $z = c$. CO2-App (16)

Or

(b) Verify Green's theorem for $\int_C x^2 dx + xy dy$, where C is bounded by x = 0, x = a, y = 0, y = a — CO2 -App (16)

18. (a) (i) Find the image of $|z - 3i| = 3$ under the transformation $w = \frac{1}{z}$ — CO3-App (8)

(ii) If f(z) = u +iv is a regular function of z in a domain D the following relation hold in D. $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$ — CO3-App (8)

Or

(b) (i) Find the image of $|z - 1| = 1$ under the transformation $w = \frac{1}{z}$ — CO3-App (8)

(ii) If f(z) is analytic whose real part is constant must itself be a constant. — CO3-App (8)

19. (a) (i) Evaluate $f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ by using Cauchy's Integral formula where C is $|z| = 3$ — CO4-App (8)

(ii) Expand $\frac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the region $2 < |z| < 3$ — CO4-App (8)

Or

(b) Using Contour integration, to prove $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a+b}$ $a > b > 0$ — CO4-App (16)

20. (a) (i) Solve the differential equation $\frac{d^2y}{dt^2} + y = \sin 2t$; $y(0) = 0$; $y'(0) = 0$ by using Laplace transform method. — CO5-App (8)

(ii) Find the inverse Laplace Transform of $\frac{s+3}{(s+1)(s^2 + 2s + 3)}$ — CO5-App (8)

Or

(b) CO5-App (8)

(i) Find the Laplace transform of $f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$

CO5-App (8)

(ii) Solve by using convolution theorem $L^{-1}\left[\frac{s}{(s^2 + a^2)(s^2 + b^2)}\right]$