Reg. No. : | | | | | | | | | | | | |

Question Paper Code: U1M01

M.E. DEGREE EXAMINATION, DEC 2025

Computer Science and Engineering (DATA SCIENCE)

21PMA101 – MATHEMATICAL FOUNDATIONS OF DATA SCIENCE

(Regulations 2021)

(Statistical Table can be provided)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART - A (5 x 20 = 100 Marks)

1.	(a)	(i) Using Jacobian Matrix Evaluate $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$.	CO1-App	(10)
		(ii) Evaluate the Eigen values and Eigen Vectors of $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.	CO1-App	(10)
		Or		
	(b)	(i) Show that s={(1,2,1,),(2,1,0),(1,-1,2) } Form a Basis.	CO1-App	(10)
		(ii) Show that the vectors $v_1 = (0,1,-2), v_2 = (1,-1,1), v_3 = (1,2,1)$ are Linearly Independent	CO1-App	(5)
		(ii) If x=(3,2,-4) ,y=(2,-5,6) ,z=(-2,1,4,) then Evaluate <x,y>,<3x,y>,<x+y,z>, ||x||.		(5)
2.	(a)	The Density Function of a random variable X is defined by f(x)=Ax(2-x) ,0$\leq x \leq 2$. Find i) the value of A ii) mean iii) variance iv) n^{th} moment about origin.	CO2-App	(20)
		Or		

(b) (i) A Random Variable X has the following probability distribution — CO2-App (10)

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find

i) 'k'

ii) $P(X<6)$, $P(X\geq 6)$ & $P(1.5<X<4.5/X>2)$

iii) If $P(X\leq k)>\frac{1}{2}$ find the minimum value of 'k'

iv) Distribution function of x v) Mean vi)Variance

(ii) The contents of urns I,II,III are follows. 1 White ,2 Black and 3Red Balls, 2 White,1black and 1 Red Balls and 4 white ,5 Black and 3 Red balls respectively. One urn is chosen at Random and 2balls are drawn from it they Happen to be white and red. What is the Probability that they come from urns I,II and III? — CO2-App (10)

3. (a) (i) If f(x_1, x_2) is the Bivariate Normal density Function show that f(x_1/x_2) is $N(\mu_1+\frac{\sigma_1}{\sigma_2}(x_2-\mu_2), \sigma_{11}-\frac{{\sigma_{12}}^2}{\sigma_{22}})$. — CO3-App (10)

Let X_1, X_2, X_3, X_4 be Independent and Identically Distributed 3×1 Random Vectors $\mu = \begin{matrix} 3 \\ -1 \\ 1 \end{matrix}$ and $\varepsilon = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.Find the Mean Vector and Covariance matrices for each of the two linear combinations of Random Vectors $\frac{1}{2}X_1+\frac{1}{2}X_2+\frac{1}{2}X_3+\frac{1}{2}X_4$. — CO3-App (10)

Or

(b) Fit a normal curve using the given data — CO3-App (20)

X	2	4	6	8	10
y	1	4	6	4	1

4. (a) The marks obtained by 10 students in Mathematics and Statistics are given as — CO4-App (20)

Mathematics	25	24	35	32	31	26	29	38	34	32
Statistics	43	46	49	41	36	32	31	30	33	39

Find (i) the two regression equation II) correlation coefficient

ii) when X=40 find Y.

Or

(b) (i) Fit the curve second degree parabola Y=$a + bx + cx^2$ for the following data using method of least square CO4-App (10)

X	0	1	2	3	4
Y	1	18	1.3	2	6.3

(ii) Fit the curve y=ax+b for the following data ,Also Estimate the value of Y at X=70. CO4-App (10)

X	71	68	73	69	67	65	66	67
Y	69	72	70	70	68	67	68	64

5. (a) Calculate the Population Principal components of the Random Variables $X_{1,}X_2, and\ \ X_3$ have the covariance Matrix CO5-App (20)

$$E = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Or

(b) The Random Vector $X^1 = [X_{1,}X_2, X_3\ , X_4\ ]$ and mean Vector $\mu^1 X = [4,3,2,1]$,Covariance Matrix CO5-App (20)

$$\varepsilon_X = \begin{pmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{pmatrix}$$

Partition X as X=$\frac{\frac{\frac{x_1}{x_2}}{x_3}}{x_4}$ =$\frac{x^{(1)}}{x^{(2)}}$

A=[1 2] ,B=$\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$

Consider the Linear Combination A$x^{(1)}$and B$x^{(2)}$.Find

(i) E($x^{(1)}$) (ii) E(A$x^{(1)}$) (iii) Cov($x^{(1)}$) (iv) Cov($Ax^{(1)}$)

(v) E($x^{(2)}$) (vi) E($Bx^{(2)}$) (vii) Cov($x^{(2)}$) (viii) Cov($Bx^{(2)}$)

(ix) Cov($x^{(1)}, x^{(2)}$) (x) Cov($Ax^{(1)}, Bx^{(2)}$) .