A   Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐☐

Question Paper Code: U2M02

B.E./B.Tech. DEGREE EXAMINATION, NOV 2025

Second Semester

Mechanical Engineering

21UMA202 - CALCULUS, FOURIER SERIES AND NUMERICAL METHODS

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Gauss Seidel iteration converges if the coefficient matrix is _____ dominant.   CO6-U

   (a) Squarely   (b) logically   (c) diagonally   (d) symmetrically

2. Newton's method also called _____ method.   CO6-U

   (a) tangents   (b) slope   (c) secants   (d) false

3. The complementary function of $(4D^2 - 3D - 1)y = 2 \sin 2x$ is _____.   CO2-App

   (a)$Ae^x + Be^{-\frac{x}{4}}$ (b) $Ae^{-x}+Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d)Ae^x+Be^{4x}

4. $\frac{1}{D^2}(\cos x) = $ _____   CO2-App

   (a) sin x   (b) - cos x   (c) cos x   (d) tan x

5. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is _____.   CO3- App

   (a) 4   (b)– 4   (c)– 3   (d) 0

6. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is _____ .   CO3- App

   (a) $2\vec{i} + 2\vec{j} + \vec{k}$ (b)$2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3   (d) 9

7. cos x is a periodic function with period ----------.   CO4-App

   (a) π   (b) 2 π   (c) π/3   (d) 2 π/3

8. If $f(-x) = -f(x)$, then $f(x)$ is said to be an _____.   CO6-U

   (a) Odd Function   (b) Even Function   (c) Periodic function   (d) Self Reciprocal

9. Convolution theorem on Fourier Transform is $F[f(x)*g(x)]=$ CO6-U

   _____

   (a) $F(s).G(s)$ (b) $f(s).g(s)$ (c) $F(s)*G(s)$ (d) $f(s)*g(s)$

10. $F_s[e^{-ax}] = $ _____   CO5-U

   (a) $\sqrt{\dfrac{2}{\pi}}\dfrac{s}{s^2+a^2}$ (b) $\sqrt{\dfrac{2}{\pi}}\dfrac{a}{s^2+a^2}$ (c) $\sqrt{\dfrac{2}{\pi}}\dfrac{a^2}{s^2+a^2}$ (d) $\sqrt{\dfrac{2}{\pi}}\dfrac{s^2}{s^2+a^2}$

<div align="center">PART – B (5 x 2= 10Marks)</div>

11. What do you mean by diagonally dominant?   CO6-U

12. Find the Particular Integral of $(D^2 - 2D + 1)y = \cosh x$ CO2-App

13. Show that $\nabla(r^n) = nr^{n-2}\vec{r}$.   CO3-App

14. Find b_n in the Fourier series of $f(x) = |\cos x|$ in $(0, 2\pi)$.   CO4-App

15. Find the Fourier Sine Transform of $f(x) = e^{-x}, x>0$ CO5-App

<div align="center">PART – C (5 x 16= 80Marks)</div>

16. (a) (i) Solve the equation $e^x - 3x = 0$ by iteration method.   CO1-App   (8)

   (ii) Solve $27x + 6y - z = 85$, $6x + 15y +2z = 72$, $x + y + 54z =110$ CO1- App   (8)
   by Gauss Jacobi Method.

<div align="center">Or</div>

   (b) (i) Using Power method find numerically largest Eigen value of   CO1- App   (8)

$$\begin{pmatrix} 9 & 1 & 8 \\ 7 & 4 & 1 \\ 1 & 7 & 9 \end{pmatrix}$$

   (ii) Solve $27x + 6y - z = 85$, $6x + 15y +2z = 72$, $x + y + 54z =110$ CO1- App   (8)
   by Gauss Seidel method

17. (a) (i) Using method of variation of parameters solve   CO2-App   (8)
   $(D^2 + a^2)y = \sec ax$

   (ii) Solve $\left(D^2 + 2D + 2\right)y = \cos 2x$ CO2-App   (8)

<div align="center">Or</div>

<div align="center">2</div>

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(b) (i) Solve $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$ CO2 -App   (8)

(ii) If the population of a country double in 50 years, in how   CO2 -App   (8)
many years will it triple under the assumption that the rate of
increase of proportional to the number of inhabitants?

18. (a) Verify G.D.T for $\iint_s \vec{F}.\hat{n}\ ds$ where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ over the   CO3-App   (16)

cylindrical region $x^2 + y^2 = 4$, $z = 0$ and z = 3

Or

(b) (i) Prove that $\bar{F} = (x^2 + xy^2)\bar{i} + (y^2 + x^2y)\bar{j}$ is irrotational   CO3-App   (8)
vector and find the Scalar potential such that $\bar{F} = \nabla\emptyset$.

(ii) Evaluate Green's theorem for $\int (x^2 - y^2)dx + 2xydy$, where   CO3-App   (8)
C is bounded by $x = 0, x = a, \ y = 0 \ and \ \ y = b$

19. (a) Express $f(x) = (\pi - x)$ as a Fourier series of period 2π in the   CO4-App   (16)
interval $0 < x < 2\pi$. Hence deduce the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$$

Or

(b) (i) The table of values of the function $y = f(x)$ is given below: CO4-App   (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent $f(x)$
in terms of x.

(ii) Find the half range sine series for $f(x) = x^2$ in $(0, l)$ CO4-App   (8)

20. (a) Find the Fourier Transform of the function $f(x)$ defined by   CO5-App   (16)

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1 \end{cases}$$ Hence Prove that :

$$\int_0^\infty \frac{\sin t - t\cos t}{t^3} dt = \frac{\pi}{4} .$$

Or

(b)   (i) Find Fourier Cosine and Sine Transform of e^{-ax}.   CO5- App   (8)

(ii)   Evaluate $\displaystyle\int_{0}^{\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$ CO5- App   (8)

U2M02