**Reg. No. :** | | | | | | | | | | | | |

# Question Paper Code: R3M22S

B.E./B.Tech. DEGREE EXAMINATION, NOV 2025

Third Semester

Computer Science and Engineering

R21UMA322-PROBABILITY, QUEUEING THEORY AND NUMERICAL METHODS

(Common to IT, IOT & CYBER SECURITY)

(Regulations R2021)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

| | | |
|---|---|---|
| 1. | Define Exponential distribution | CO6 U |
| 2. | If $P(X = x) = \frac{k}{x}, x = 1,2,3,5$ represents p.m.f , compute the value of 'K' | CO1 App |
| 3. | What is Queuing Theory? | CO6 U |
| 4. | Explain Kendall's Notation (a/b/c): (d/e) of a queueing model | CO6 U |
| 5. | Write down the Normal Equations of the curve $y = ae^{bx}$ | CO3 App |
| 6. | Write the normal equations for fitting a parabola $y = ax^2 + bx + c$ | CO3 App |
| 7. | Using Newton's Raphson for two steps $e^x = 3x$ by taking $x_0 = 1.5$ | CO4 App |
| 8. | Write the condition of convergence of Newton's method | CO4 App |
| 9. | Using Euler's method Compute y(0.1) given $\frac{dy}{dx}$ =1+ $y^2$, y(0) =0 | CO5 App |
| 10. | Write down the Taylor's series formula. | CO6 U |

PART – B (5 x 16= 80 Marks)

11. (a) A RV X has the following distribution CO1- App (8)

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(X) | 0 | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2+k$ |

(i) Find the value of 'k'

(ii) Find $P(1.5 < X < 4.5/\ X > 2)$

(iii) If $P(X \geq k) > \frac{1}{2}$ find the minimum value of 'k' & Find the distribution function of X

(ii) The two dimensional RV (X,Y) has the density function CO1- App (8)
$p(x,y) = k(2x+4y),\ x = 1,3,4,\ y = 1,2,4$ Find i) The marginal distribution function of X and Y ii) Find the conditional distributions

Or

(b) (i) Compute the moment generating function of Exponential distribution and hence find it's mean and variance. CO1- App (8)

(ii) Calculate the Correlation coefficient for the following data CO1-App (8)

| X | 54 | 55 | 51 | 50 | 48 | 44 | 56 | 57 |
|---|---|---|---|---|---|---|---|---|
| Y | 175 | 177 | 178 | 165 | 168 | 166 | 160 | 164 |

12. (a) (i) Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 11 minutes, and the service time is an exponential random variable with mean 9 minutes. Identify the Model , Compute the following i) the average number of customers in the shop $L_s$ ii) the average time a customer spends in the shop $W_s$ iii) the average number of customers in the queue $L_q$ iv) the probability that the server is idle. CO2- Ana (8)

(ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 15 patients. Examination time per patient is exponential with mean rate of 20 per hour. Identify the Model , Compute i) the effective arrival rate at the clinic. ii)the probability that an arriving patient will not wait? iii) the expected waiting time until a patient is discharged from the clinic? CO2- Ana (8)

Or

(b) (i) Customers arrive at a one-man barbershop according to a Poisson process with a mean inter - arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. If an hour is used as a unit of time, then i) What is the probability that the customer need not wait for a haircut? ii) What is the expected number of customers in the barbershop and in the queue? iii) How much time can a customer expect to spend in the barbershop? iv)Find the average time that the customers spend in the queue? v) What is the probability that there will be 6 or more customers waiting for service? CO2-Ana (8)

(ii) In a car - wash service facility, cars arrive for service according to Poisson distribution with mean 5 per hour. The time for washing and cleaning each car has exponential distribution with mean 12 minutes per car. The facility cannot handle more than one car at a time and has a total of 5 parking spaces. Find the effective arrival rate b) What is the probability that an arriving car will get service immediately upon arrival? c) Find the expected number of parking spaces occupied CO2- Ana (8)

13. (a) (i) Applying method of moments, fit a straight line to the following data CO3- App (8)

| X | 1 | 3 | 5 | 7 | 9 | 11 |
|---|---|---|---|---|---|---|
| Y | 2.4 | 5.3 | 8.3 | 10.2 | 11.5 | 12.9 |

(ii) Applying group average method fit a second degree parabola $y = ax^2 + bx + c$ for the following data CO3-App (8)

| X | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 11 |
|---|---|---|---|---|---|---|---|---|---|
| Y | 15 | 19 | 29 | 35 | 42 | 50 | 59 | 68 | 89 |

Or

(b) (i) Applying group average method fit the curve $y = ax^b$ with the following data: CO3- App (8)

| X | 2 | 4 | 5 | 7 | 10 | 15 | 18 |
|---|---|---|---|---|---|---|---|
| Y | 8.6 | 9.7 | 10 | 10.6 | 11.3 | 12.1 | 13 |

(ii) Applying least square method fit a straight line $y = a + bx$ for the following data CO3-App (8)

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Y | 17.9 | 17.3 | 16.6 | 16 | 15.3 | 14.7 | 14 | 13.4 | 12.7 | 12 |

14. (a) (i) Compute the real positive root of $x log_{10} x = 12.12$ by Newton's Raphson Method. Correct to 4 decimal places — CO4 -App — (8)

(ii) Applying Power method compute numerically largest Eigen value of $\begin{pmatrix} 1 & 5 & 3 \\ 2 & -1 & 0 \\ 4 & 0 & 8 \end{pmatrix}$ by taking $X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ — CO4-App — (8)

Or

(b) (i) Solve 27x + 6y – z = 85, 6x + 15y +2z = 72, x + y + 54z =115 by Gauss Seidel method — CO4- App — (8)

(ii) Compute the real positive root of $x^5 - 3x - 45 = 0$ by Iterative Method, Correct to 4 decimal places — CO4-App — (8)

15. (a) (i) Using R.K Method of 4th order, solve $\frac{dy}{dx} = \frac{y}{1 + x^2}$ with y ( 0 ) = 1, Compute y ( 0.1 ) by taking h=0.1 — CO5-App — (10)

(ii) Given $\frac{dy}{dx} = 3y + 5x$ with y (0) = 7.5, Compute y approximately for x= 0.5 by Euler's method in five steps — CO5-App — (6)

Or

(b) (i) Given $\frac{dy}{dx} = 2e^x - y$ , y(0) =2 , y(0.1) = 2.010 , y(0.2) =2.040, y(0.3) = 2.090 Evaluate y( 0.4 ) By Adams – Bashforth Method — CO5-App — (8)

(ii) Using Milen's method find y(0.4) given $y' = y - \frac{2x}{y}$, given y (0) = 1, y(0.1) = 1.0959, y(0.2) = 1.1841, y(0.3) = 1.2662 — CO5-App — (8)