A   Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐☐

Question Paper Code: U5M01

B.E./B.Tech. DEGREE EXAMINATION, NOV 2025

Fifth Semester

Artificial Intelligence & Data Science

21UMA501 - LINEAR ALGEBRA

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The dim(R^3)is _____   CO6- U

    (a) 1   (b) 2   (c) 3   (d) 0

2. The trivial subspaces of a vector space V are _____   CO6-U

    (a) {0}   (b) V   (c) W   (d) {0}and V

3. If T: V→W be linear transformation then T(0) = _____   CO6- U

    (a) 0   (b) 1   (c) 2   (d) 3

4. In a linear transformation T: V→W the kernel of T is a subspace of   CO6- U
    _____

    (a) V   (b) W   (c) both V and W   (d) none of these

5. $\langle x, x \rangle = 0$ if and only if _____   CO6- U

    (a) $x = 1$ (b) $x \neq 1$ (c) $x = 0$ (d) $x \neq 0$

6. In a vector space V , if $\langle x, y \rangle = \langle y, z \rangle$ then _____   CO6- U

    (a) $y = z$ (b) $y \neq z$ (c) $y = -z$ (d) none of these

7. The Hermitian Matrices containing some _____ numbers.   CO6- U

    (a) Real   (b) Imaginary   (c) Natural   (d) None of these

8. All Eigen values of a _____ matrix are real   CO6- U

    (a) Symmetric   (b) Skew Symmetric   (c)Hermitian   (d) None of these

9. A square matrix A is said to be ------------- if the determinant value of A   CO6- U
   is zero.

   (a) Singular   (b) Non - Singular   (c) Symmetric   (d) none of these

10. Linear Programming deals with the ------------ of a function of decision   CO6- U
    variable.

    (a) Optimization   (b) Formulation   (c) Technique   (d) none of these

<center>PART – B (5 x 2= 10Marks)</center>

11. Prove that W={(a,0,0)/a $\in$ R} is a subspace of R^3 CO1-App

12. Define Rank nullity theorem   CO6-U

13. Find the norm of $(1,2,3)$ in $V_3(R)$ with standard inner product.   CO3 -App

14. Let A= $\begin{pmatrix} 11 & -6i \\ 4i & 1 \end{pmatrix}$ Find the Characteristic equation of A   CO6-U

15. State whether y = tanx can be expanded as a Fourier series.   CO5- App

<center>PART – C (5 x 16= 80Marks)</center>

16. (a) (i) Verify the vectors (1,2,0) , (2,3,0) ,(8,13,0) in R^3 is a basis of R^3 CO1- App   (8)
        (ii) In R^3 determine whether (1,7,-4) is expressed as a linear   CO1- App   (8)
        combination of (1,-3,2) and (2,-1,1)
    <center>Or</center>
    (b) (i) Find the dimension of the subspace spanned by the vectors   CO1 -App   (8)
        (1,2,-3) ,(0,0,1) ,(-1,2,1) in $V_3(R)$
        (ii) Write the Vector V = (1,-2,5) as a Linear combination of the   CO1 -App   (8)
        Vectors e$_1$ = (1,1,1), e$_2$ = (1,2,3) and e$_3$ = (2,-1,1) in R^3

17. (a) (i) If T: R^2 → R^2 be linear transformation defined by   CO2- App   (8)
        $T(a_1, a_2) = (a_1 + a_2, a_1)$ then find nullity(T) ,rank(T), Is T one-to-
        one? Is T onto? Also check the rank nullity theorem.
        (ii) Find the matrix of the linear transformation T: R^2 → R^2 defined   CO2- App   (8)
        by T(a,b) =(2a-3b,a+b) for the standard basis of R^2
    <center>Or</center>
    (b) T: V$_1$(R) →V$_3$(R) is defined by T(a$_1$) = (a$_1$, 2a$_1$^{2}, a$_1$^{3}) Verify   CO2- App   (16)
        whether T is a linear transformation

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18. (a) (i) Show that the following function defines an inner product on $V_2(R)$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$ and $\langle x, y \rangle = 6x_1y_1 + 7x_2y_2$ — CO3- App (8)

(ii) If $x = (2, 1 + i, i)$ and $y = (2 - i, 2, 1 + 2i)$ then verify Schwarz's inequality. — CO3- App (8)

Or

(b) Apply Gram-Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1 = (1, -1, 0)$, $v_2 = (2, -1, -2)$ and $v_3 = (1, -1, 2)$ — CO3- App (16)

19. (a) Construct a QR factorization for a matrix — CO4- App (16)
$$A = \begin{pmatrix} -4 & 4 & 2 \\ 4 & -4 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

Or

(b) Determine the Cholesky Decomposition for — CO4- App (16)
$$A = \begin{pmatrix} 16 & -3 & 5 & -8 \\ -3 & 16 & -5 & -8 \\ 5 & -5 & 24 & 0 \\ -8 & -8 & 0 & 21 \end{pmatrix}$$

20. (a) From the following information state whether the condition of the child is associated with the condition of the house — CO5- App (16)

Condition of child	Clea	Dirty	Total
Clean	69	51	120
Fairly clean	81	20	101
Dirty	35	44	79
Total	185	115	300

Or

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(b) The Savings bank account of a customer showed an average balance   CO5- App   (16)
of Rs. 150 and a SD of Rs 50 assuming that the account balances are
normally distributed.

       i)   What percentage of account is over Rs. 200 ?

       ii)   What percentage of account is between Rs.120 and Rs. 170 ?

       iii)   What percentage of account is less than Rs. 75 ?

             $(0 < z < 04 = 0.3811, \; 0 < z < 0.6 = 0.2257,$

$0 < z < 1.5 = 0.4332, \; 0 < z < 1 = 0.3143 \,)$

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