A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R2M02

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Mechanical Engineering

R21UMA202 - CALCULUS, FOURIER SERIES AND NUMERICAL METHODS

(Regulations R2021)

(Common to Chemical Engineering)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Iteration method converges if $|g'(x)|$ ___________ CO6- U

(a) >1 (b)<1 (c)=0 (d) >0

2. When Gauss Elimination method is used to solve AX=B, A is transferred in a ________ matrix CO6- U

(a) lower triangular (b) upper triangular (c) square (d) zero

3. The auxiliary equation of the equation $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ is _____ CO2-App

(a) $m^2 - 4m + 5 = 0$ (b) $m^2 + 3m - 2 = 0$ (c) $m^2 + 3m + 2 = 0$ (d) $2m^2 + 5m - 7 = 0$

4. The solution of $(D^3 + D^2 - D - 1)y = 0$ is _____ CO2-App

(a) $Ae^x + Bxe^x + Cx^2e^x$ (b) $(Ax + B)e^x + C e^{-x}$

(c) e^{-x} + (cos 2x + i sin 2x) (d) $(Ax + B)e^{-x} + C e^x$

5. Gauss Divergence theorem is a relation between ______ CO6-U

(a) line and volume integral (b) line integral and surface integral

(c) surface integral and volume integral (d) volume integral and line integral

6. If we take z = log x and $\theta = \frac{d}{dx}$, then $x^2\frac{d^2y}{dx} =$ _____ CO6-U

(a) $(\theta-1)y$ (b) $\theta(\theta-1)y$ (c) $\theta^2 y$ (d) $(\theta^2 - 1)y$

7. If a function f(x) is even, its Fourier expansion contains only ------- terms CO6- U

(a) First harmonic (b) Second harmonic (c) Third harmonic (d) Fourier Coefficients

8. The Fourier constant b_n in $(-\pi,\pi)$ for x sin x is _____ CO4-App

(a) x^2 (b) 3x (c) 0 (d) 1

9. If f(x) is an odd function then $\int_{-a}^{a} f(x)dx =$____ CO6-U

(a) 0 (b) $\frac{1}{2}\int_0^a f(x)dx$ (c) $2\int_0^a f(x)dx$ (d) $\int_0^a f(x)dx$

10. In Modulation property, F[f(x) cosax] = _______ CO6-U

(a) $\frac{1}{2}[F(s+a)-F(s-a)]$ (b) $\frac{1}{2}[F(s+a)+F(s-a)]$

(c) $[F(s+a)-F(s-a)]$ (d) $F(s+a)+F(s-a)$

PART – B (5 x 2= 10Marks)

11. State Newton's Iterative formula CO6- U

12. Find the Particular Integral of $(D^2 + 1)$ y = sin x CO2 App

13. Is the position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ irrotational? Justify. CO3 App

14. What you meant by Harmonic analysis? CO6- U

15. Find the Fourier transform of $f(x) = \begin{cases} 1 & ,|x| < 2 \\ 0 & ,|x| > 2 \end{cases}$ CO5 App

PART – C (5 x 16= 80Marks)

16. (a) (i) Using Power method find numerically largest Eigen value of $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$. CO1 App (8)

(ii) Solve 28x+4y-z = 32: x+3y+10z = 24 ; 2x+17y+4z = 35 by Gauss Seidal method CO1 App (8)

Or

(b) (i) Solve for a positive root of 3x – cos x - 1 =0 by Newton's Raphson method . CO1 App (8)

(ii) Solve 4x + 2y +z = 14, x + 5y -z = 10, x + y + 8z =20 by Gauss Elimination method CO1 App (8)

17. (a) (i) Using method of variation of parameters solve $(D^2 + 4)y = 4\tan 2x$. CO2 App (8)

(ii) Solve $(x^2D^2 + 4xD + 2)y = x + \frac{1}{x}$ CO2 App (8)

Or

(b) (i) Using method of variation of parameters solve $(D^2 + 4)y = \cot 2x$. CO2 App (8)

(ii) Solve $(x^2D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$ CO2 App (8)

18. (a) (i) Prove that $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2 y)\vec{j}$ is irrotational vector and find the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO3 App (8)

(ii) Evaluate Stoke's theorem for $\int (x^2 - y^2)dx + 2xydy$, where C is bounded by $x-0, x=a, \ y=0 \ and \ y=b$ CO3 App (8)

Or

(b) Verify Green's theorem for $\int (x^2 + y^2)dx - 2xydy$, where C is bounded by $x \pm a, \ y=0 \ and \ y=b$ CO3 App (16)

19. (a) (i) Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series of period 2π in the internal $0 < x < 2\pi$. CO4 App (8)

(ii) The table of values of the function $y = f(x)$ is given below: CO4 App (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.8	0.3	0.5	2.6	1.3	1.7	1.8

Find a Fourier series up to the third harmonic to represent f(x) in terms of x.

Or

(b) (i) Find the Half range cosine series for $f(x) = x$ in $(0, \pi)$. Deduce that $\sum_{n=odd}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{96}$. CO4 App (8)

(ii) Find the Fourier series of $f(x) = x+x^2$ in $(-\pi, \pi)$ of periodicity 2π. Hence deduce that the value of the sum $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. CO4 App (8)

20. (a) (i) Find the Fourier sine & cosine transform of e^{-ax}. CO5 App (8)

(ii) Evaluate CO5 App (8)

$$\int_0^\infty \frac{dx}{(x^2+25)^2}$$

Or

(b) (i) Find the Fourier cosine transform of x^{n-1}, 0 <n <1, x>0 and hence deduce that $\frac{1}{\sqrt{x}}$ is self-reciprocal under the Fourier cosine transform. CO5 App (8)

(ii) Evaluate CO5 App (8)

$$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$$