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B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Third Semester

Computer Science and Engineering

19UMA322-Probability, Queueing Theory and Numerical Methods

(Regulation 2019)

(Common to Information Technology)

Duration: Three hours

(a)

Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10Marks)

1. If X is the discrete random variable having the probability mass function, CO1- App then K value is .

|      | X    | 1               | 2          | 5 |          |
|------|------|-----------------|------------|---|----------|
|      | P(X) | 9k <sup>2</sup> | $k^2 + 2k$ | k |          |
| )1/5 | (b)  | -1/5            | (a)1/:     | 5 | (b) -1/5 |

- 2. A Continuous r.v has a p.d.f  $f(x) = 3x^2$ ,  $0 \le x \le 1$ , If P(X > b) = 0.05, then value of b CO1- App is
  - (a) 0.9308 (b) 0.9803 (c) 0.9830 (d) 0.9038
- 3. The relation between  $L_s \& L_q$  is

(a) 
$$L_s = \lambda L_q$$
 (b)  $L_q = \lambda L_s$  (c)  $L_q = L_s + \frac{\lambda}{\mu}$  (d)  $L_s = L_q + \frac{\lambda}{\mu}$ 

- 4. For a model (M/M/1): ( $\infty$ /FCFS)The arrival rate is 3 per hour and service rate CO2- App is 4 per hour then  $W_s$ 
  - (a) 55 Minutes (b) 65 Minutes (a) 55 Minutes (b) 65 Minutes
- 5. In method of moments ,the first moment is denoted by CO6- U (a)  $\Sigma x \Delta y$  (b)  $\Sigma y \Delta x$  (a)  $\Sigma x \Delta y$  (b)  $\Sigma y \Delta x$

CO6- U

| 6.  | num<br>method of lea                     | ber of normal equations a ast squares   | are required to fit a stra  | hight line in CO6- U |
|-----|--|---|-----------------------------|----------------------|
|     | (a) 1                                    | (b) 2   | (a) 1                       | (b) 2                |
| 7.  |  | natrix, 5, 10 are the Eiger<br>nt Eigen value                                     | n values, trace of matrix i | s equal to 3 CO6- U  |
|     | (a) 12                                   | (b) -12   | (a) 12                      | (b) -12              |
| 8.  | Iteration met                            | hod converges if $ g^1(x) $   |                             | CO6- U               |
|     | (a) >1                                   | (b)<1   | (a) >1                      | (b)<1                |
| 9.  | In Euler's me                            | ethod, if h is small, the method  | hod is too                  | CO6- U               |
|     | (a) fast                                 | (b)slow   | (a) fast                    | (b)slow              |
| 10. | prior v                                  | values are required to predi-   | ct the next value in Milne' | s method CO6- U      |
|     | (a) 1                                    | (b)2  | (a) 1                       | (b)2                 |
|     |  | PART – B  | (5 x 2= 10Marks)            |                      |
| 11. | A coin is to atleastonehea               | ossed thrice; Compute the   | e probability that there    | will appear CO1- App |
| 12. | Explain Kend                             | dall's Notation (a/b/c): (d/e   | e) of a queueing model      | CO6- U               |
| 13. | Write down t                             | he Normal Equations of the  | e curve $y = ae^{bx}$       | CO6- U               |
| 14. | Write the cor                            | ndition of convergence of N   | lewton's method             | CO6- U               |
| 15. | Write down t                             | he Milne's predictor and co   | prrector formula.           | CO5 U                |
|     |  | PART –  | C (5 x 16= 80Marks)         |                      |
| 16. |  | ne density function of a con  | tinuous r.v X is given by   | CO1-Ana (8)          |
|     |  | $ax \qquad 0 \le x \le 1$ $a \qquad 1 \le x \le 2$ $3a - ax \qquad 2 \le x \le 3$ |                             |                      |
|     | $f(x) = \begin{cases} \\ \\ \end{cases}$ | $a \qquad 1 \le x \le 2$  |                             |                      |
|     |  | $3a - ax$ $2 \le x \le 3$   |                             |                      |

(a). Compute the value of "a" (b). Compute the c.d.f of X

(ii) Using the p.df of an exponential distribution, State and prove CO1-Ana (8) memoryless property

Or

## CO1 - Ana (8)

## (b) (i) A RV X has the following distribution

| X                    | 0 | 1  | 2  | 3  | 4  | 5   | 6   | 7   | 8   |
|----------------------|---|----|----|----|----|-----|-----|-----|-----|
| P(X)                 | а | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |
| P[1 < X < 5 / X > 3] |   |    |    |    |    |     |     |     |     |

i) Compute  $P(X \ge 4)$  and P[1 < X < 5 / X > 3]

ii) Compute E(X)

(ii) Calculate the Correlation coefficient for the following data CO1 -Ana

|   |         |         |     |     |     |     | 25  |     |
|---|---------|---------|-----|-----|-----|-----|-----|-----|
| Y | 11<br>0 | 12<br>0 | 124 | 130 | 136 | 122 | 140 | 143 |

17. (a) (i) Customers arrive at a watch repair shop according to a Poisson CO2 -Ana (8) process at a rate of 10 per every an hour, and the service time is an exponential random variable with mean 4 minutes. i) Compute the average number of customers in the shop  $L_s$  ii) Compute the average time a customer spends in the shop  $W_s$  iii) Compute the average number of customers in the queue  $L_q$  iv) What is the probability that the server is idle

(ii) Patients arrive at a clinic according to Poisson distribution at a CO2 -Ana (8) rate of 32 patients per hour. The waiting room does not accommodate more than 15 patients. Examination time per patient is exponential with mean rate of 20 per hour. Identify the Model , Compute i) the effective arrival rate at the clinic. ii)the probability that an arriving patient will not wait? iii) the expected waiting time until a patient is discharged from the clinic?

## Or

(b) A petrol pump station has 4 pumps. The service times follow the CO2 -Ana (16) exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. Identify the Model, Compute the following i) the Probability that an arrival would have to wait in line? ii) the average waiting time, average time spent in the system and the average number of cars in the system iii) For what percentage of time would a pump be idle on an average?

CO1 - Ana (8)

18. (a) (i) Applying least square method techniques fit a straight line CO3- App (8) y=a+bx

| Х | 0 | 3 | 5 | 6 | 8  | 10 | 12 |
|---|---|---|---|---|----|----|----|
| Y | 2 | 5 | 8 | 9 | 11 | 12 | 15 |

(ii) Applying group average method fit a second degree parabola CO3- App (8)  $y = a + bx + cx^2$  for the following data

| Х  | 1 | 2  | 3  | 4  | 5  |  |  |  |
|----|---|----|----|----|----|--|--|--|
| Y  | 5 | 12 | 26 | 60 | 97 |  |  |  |
| Or |   |    |    |    |    |  |  |  |

(b) (i) Applying method of moments fit a straight line y = ax + b CO3- App (8)

| Х | 1 | 3   | 5    | 7  |
|---|---|-----|------|----|
| Y | 4 | 8.5 | 11.5 | 15 |

(ii) Applying least square method techniques fit the curve  $y = ab^x$  CO3-App (8) with the following data:

| X | 1   | 2  | 3  | 4  | 5  |
|---|-----|----|----|----|----|
| Y | 150 | 99 | 60 | 48 | 18 |

19. (a) (i) Compute the real positive root of  $x \log_{10} x = 4.5$  by Newton's CO4-App (8) Raphson Method. Correct to 3 decimal places

(ii) Applying Power method compute numerically largest Eigen CO4-App (8)

value of  $\begin{pmatrix} 9 & 10 & 8 \\ 10 & 5 & -1 \\ 8 & -1 & 3 \end{pmatrix}$  by taking  $X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ 

Or

(b) (i) Using Gauss Seidel method, Solve 28x+4y-z = 32: x+3y+10z = CO4 -App (8) 24;

2x+17y+4z = 35

(ii) Compute the real positive root of  $3x - \cos x = 1$  by Iterative CO4 - App (8) method

20. (a) (i) Using R.K Method of 4<sup>th</sup> order, solve  $\frac{dy}{dx} = \frac{y}{1+x^2}$  with y (0) CO5- App (8) = 1, Compute y (0.1) by taking h=0.1

(ii) Given  $\frac{dy}{dx} = y + 2x$  with y (0) = 1, Compute y approximately CO5- App (8) for x=0.5 by Euler's method in five steps

Or

(b) Given  $\frac{dy}{dx} = x^3 + y$ , y(0) = 2, y(0.2) = 2.443, y(0.4) = 2.99, CO5-App (16)

y(0.6) = 3.68 Compute y(0.8) by Milne's Predictor & Corrector method