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**Question Paper Code:U2M03**

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Computer Science and Engineering

21UMA203- Differential Equations and Complex analysis

(Regulations 2021)

(Common to information technology)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- $\frac{1}{(D-m)^2} e^{mx} = \underline{\hspace{2cm}}$  CO1-App  
(a)  $x e^{mx}$  (b)  $x^2 e^{mx}$  (c)  $\frac{x^2}{2} e^{mx}$  (d)  $\frac{x^2}{m} e^{mx}$
- The complementary function of  $(4D^2 - 3D - 1)y = 2 \sin 2x$  is  $\underline{\hspace{2cm}}$  CO6-U  
(a)  $Ae^x + B e^{-x/4}$  (b)  $Ae^{-x} + B e^{5x}$  (c)  $(A+Bx)e^{2x}$  (d)  $Ae^x + B e^{4x}$
- $\text{Div } \vec{r} = \underline{\hspace{2cm}}$  CO2-App  
(a) 0 (b) 1 (c) 3 (d)  $\vec{r}$
- Divergence of vector  $x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$  at (1, 2, -3) is  $\underline{\hspace{2cm}}$  CO2-App  
(a) 8 (b) 4 (c) -3 (d) 0
- The critical point of the transformation  $w = z + \frac{1}{z}$  are  $\underline{\hspace{2cm}}$  CO3-App  
(a)  $\pm 1$  (b)  $\pm 2$  (c)  $\pm i$  (d)  $-i$
- The function  $f(z) = \frac{1}{z^2+4}$  is not analytic at  $z = \underline{\hspace{2cm}}$  CO3-App  
(a) 2 (b) -2 (c) 2i (d)  $\pm 2i$
- Simple pole is a pole of order  $\underline{\hspace{2cm}}$  CO6-U  
(a) 1 (b) 4 (c) 3 (d) -4

8.  $\int_C \frac{e^z}{z-2} dz$  where C is the unit circle with centre as origin is CO4-App

- (a) 0 (d) 1 (c) 2 (d)  $\pi$

9. The PDE obtained from  $z = (x+a)(y+b)$  is \_\_. CO5-App

- (a)  $3z = px + qy$  (b)  $py - qx = 0$  (c)  $z = pq$  (d)  $px+qy = 0$

10. The subsidiary equations of Lagrange's linear equation is --- CO5-U

- (a)  $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$  (b)  $\frac{dx}{P} + \frac{dy}{Q} + \frac{dz}{R}$  (c)  $\frac{dx}{P} - \frac{dy}{Q} - \frac{dz}{R}$  (d)  $Pp + Qq = R$

PART – B (5 x 2= 10Marks)

11. Find the Wronskian of  $y_1, y_2$  of  $y'' - 2y' + y = e^x \log x$  CO1-App

12. Compute  $\nabla \phi$ , if  $\phi = x^2 + y^2 + z^2$  at (1, -1, 1). CO2-App

13. Prove that  $u = e^x \cos y$  is harmonic function CO3-App

14. Using Cauchy's integral formula, Evaluate  $\int_C \frac{z}{z-2} dz$  where C is  $|z|=1$  CO4-App

15. Find the particular integral of  $(D^2 - 2DD' + D'^2)Z = \cos(x - 3y)$  CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve  $(D^2 + 2D + 2)y = e^{-2x} + \cos 2x$  CO1-App (8)

(ii) Using method of variation of parameters solve  $(D^2 + a^2)y = \text{Cosec } ax$  CO1- App (8)

Or

(b) (i) Solve  $(x^2 D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$  CO1- App (8)

(ii) A colony of bacteria of growing exponentially. At time  $t=0$  it has 10 bacteria in it and at time  $t = 4$  it has 2000. At what time will it have 100,000 bacteria?

17. (a) Verify Green's theorem in the XY plane for  $\int_C (3x^2 - 8y^2 dx + 4y - 6xy dy)$  where C is the boundary of the region defined by  $y = y^2, y = x^2$ . CO2-App (16)

Or

- (b) Verify Gauss divergence theorem for the vector function  $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$  over the cube bounded by  $x = 0, y = 0, z = 0$  and  $x = 1, y = 1, z = 1$  CO2-App (16)

18. (a) (i) Determine the analytic function whose real part is  $\frac{\sin 2x}{\cosh 2y - \cos 2x}$  CO3-App (8)

- (ii) Determine the image of  $|z - 2i| = 2$  under the transformation  $w = \frac{1}{z}$  CO3-App (8)

Or

- (b) (i) If  $f(z) = u + iv$  is an analytic function then Prove that  $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$  CO3-App (8)

- (ii) Determine the bilinear transformation which maps  $z = 1, i, -1$  respectively onto  $w = i, 0, -i$  CO3-App (8)

19. (a) (i) Using Cauchy's integral formula, Evaluate  $\int_C \frac{z+1}{(z-3)(z-1)} dz$  where C is the circle  $|z| = 2$  CO4-App (8)

- (ii) Evaluate  $f(z) = \frac{7z-2}{z(z+1)(z-2)}$  in Laurent's series valid in the region  $1 < |z+1| < 3$  CO4-App (8)

Or

- (b) Using Contour integration Prove that  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{a+b}$   $a > b > 0$  CO4-App (16)

20. (a) (i) Solve :  $(D^2 - 3DD' + 2D'^2)Z = e^{3x-2y} + \sin(3x+2y)$  CO5-App (8)
- (ii) Solve :  $x(y-z)p + y(z-x)q = z(x-y)$  CO5-App (8)

Or

- (b) A tightly String with fixed end points  $x=0$  and  $x=l$  is initially at rest in its equilibrium position. If its set vibrating giving each point at velocity  $\lambda(l-x^2)$ . Determine the displacement function  $y(x,t)$ . CO5-App (16)

