Reg. No. : | | | | | | | | | | | | |

Question Paper Code: R2M03

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Computer Science and Engineering

R21UMA203- DIFFERENTIAL EQUATIONS AND COMPLEX ANALYSIS

(Regulations R2021)

(Common to IT, Cyber Security & IOT Engineering Branches)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Complementary function of $(D-1)^3 y = 2x$ CO1- App

(a) $(A + Bx + cx^2)e^x$ (b) $(A + Bx + cx^2)e^{-x}$

(c) $(A + Bx + cx^2)$ (d) $(Ax + Bx^2 + cx^3)e^x$

2. The complete solution of $(x^2D^2 - 3xD - 5)y = 0$ is ___ CO1- App

(a) $Ae^{-z} + B e^{5z}$ (b) $Ae^{z} + B e^{5z}$ (c) $Ae^{z} + B e^{-5z}$ (d) $Ae^{-z} + B e^{-5z}$

3. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is _____ CO2- App

(a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3 (d) 9

4. IF $\vec{F}$ is Solenoidal then $(\nabla . \vec{F} + 2)$ CO2- U

(a) 1 (b) 2 (c) 3 (d) 0

5. The function $f(z) = \frac{1}{z^2+4}$ is not analytic at $z =$ ____ CO3- App

(a) 2 b) -2 c) 2i d) $\pm 2i$

6. Find the fixed points of $w = \frac{4z - 4}{z}$ CO3- App

(a) i (b) 2 (c) 3i (d) 0

7. The value of $\int_C \frac{dz}{z^2} = 0$ where C is _____ CO4- App

(a) $|z| = 1$ (d) $|z - 2| = 1$ (c) $|z| = 2$ (d) $|z - 2| = 3$

8. The residue of $f(z) = \frac{4}{z^3(z-2)}$ at its simple pole is _____ CO6- U

(a) $\frac{4}{7}$ (d) $\frac{3}{4}$ (c) $\frac{1}{7}$ (d) $\frac{3}{4}$

9. The PDE obtained from z = (x+a)(y+b) is __. CO5- App

(a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

10. $Au_{xx}+Bu_{xy}+Cu_{yy}+Du_x+Eu_y+Fu = f(x, y)$ is parabolic _____. CO5- U

(a) $B^2-4AC<0$ (b) $B^2-4AC= 0$ (c) $B^2-4AC>0$ (d) $B^2-4AC\neq0$

PART – B (5 x 2= 10Marks)

11. Compute the particular Integral $(D^2+1)y = x^2$ CO1-App

12. If $\vec{F} = (4x+3y)\vec{i} + (5y-2z)j + (x+3\lambda z)\vec{k}$ is Solenoidal then calculate the value of λ CO2-App

13. Find the image of the circle $|z| = 3$ under the transformation w = 2z CO3-App

14. Using Cauchy's integral formula , Evaluate $\int_c \frac{z}{z-2}dz$ where C is $|z| = 3$ CO4-App

15. Find the particular integral of $\left(D^2 + DD' + D'^2\right)Z = e^{2x+y}$ CO5-App

PART – C (5 x 16= 80 Marks)

16. (a) (i) Solve CO1-App (8)

$(x^2D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$

(ii) Solve the differential equation CO1-App (8)

$[(x+1)^2D^2 + (x+1)D + 4]y = \cos[\log(x+1)]$

Or

(b) (i) Solve the differential equation $(D^2 + 5D + 4)y = e^{2x} + x^2$ CO1-App (8)

(ii) Using method of variation of parameters solve CO1-App (8)
$(D^2 + a^2)y = \tan ax$

17. (a) Verify Gauss divergence theorem for the vector function $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ and $x = 1, y = 1, z = 1$ CO2-App (16)

Or

(b) Verify Green's theorem for $\int_C x^2\,dx + xy\,dy$ where c is the curve in the XY plane given by $x = 0, y = 0, x = a, y = a$. CO2-App (16)

18. (a) (i) Determine the analytic function whose imaginary part is $e^x(x\cos y - y\sin y)$ CO3-App (8)

(ii) Determine the bilinear transformation which maps $z = -1, 0, 1$ onto $w = -5, -1, 3$ CO3-App (8)

Or

(b) (i) Determine the analytic function for which $3u + 2v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ CO3-App (8)

(ii) Determine the image of the infinite strips CO3-App (8)

(a) $\frac{1}{4} < y < \frac{1}{2}$

(b) $0 < y < \frac{1}{2}$ under the mapping $w = \frac{1}{z}$

19. (a) (i) Cauchy's Residue theorem , Evaluate $\int_c \frac{2z+7}{(z-3)(z-1)(z+2)}dz$ where C is the circle $|z| = 4$ CO4-App (8)

(ii) Evaluate $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in Laurent's series valid in the region $1 < |z+1| < 3$ CO4-App (8)

Or

(b) (i) Using Contour integration, evaluate $\int_0^{\pi} \frac{1}{5 - 2\sin\theta}\, d\theta$ CO4-App (10)

(ii) Using Cauchy's Residue theorem Evaluate $\int_C \frac{5-2z}{z(z-2)(z-1)}dz$ where C is the circle $|z| = 4$ CO4-App (6)

20. (a) If a string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $V_0 \sin^3\left(\frac{\pi x}{l}\right), 0 < x < l$. Determine the displacement function y(x,t). CO5-App (16)

Or

(b) (i) Solve : $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ CO5-App (8)

(ii) Solve : $(3z - 4y)p + (4x - 5z)q = 5y - 3x$ CO5 -App (8)