**Reg. No:** | | | | | | | | | | | | |

**Question Paper Code :R3M23**

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Third Semester

Electronics And Communication Engineering

R21UMA323- NUMERICAL ANALYSIS AND LINEAR ALGEBRA

(Regulations R2021)

Duration: Three hours | Maximum: 100 Marks

PART A - (10 x 1 = 10 Marks)

1. Trapezoidal rule is so called, because it approximates the integral by the sum of _____trapezoids CO6-U

   (a) n (b) n+1 (c) n-1 (d) 2n

2. In Simpson's 3/8 rule the number of subintervals should be_____ CO6-U

   (a) multiple of 1 (b) multiple of 2 (c) multiple of 3 (d) All of these

3. Taylor Series method will be very useful to give some _____ values for RK, Milne's and Adam's methods CO6-U

   (a) initial (b) final (c) intermediate (d) two

4. _____ prior values are required to predict the next value in Milne's method CO6-U

   (a) 1 (b) 2 (c) 3 (d) 4

5. PDE of second order, if $B^2 - 4AC < 0$ then CO6-U

   (a) parabolic (b) elliptic (c) hyperbolic (d) Non homogeneous

6. Bender-Schmidt recurrence equation is valid if ƛ□= CO6-U

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

7. In a vector space V, for every $x, y \in V$ then the property x+y=y+x is known as _____ CO6-U

   (a) commutative (b) associative (c) identity (d) inverse

8. In a linear transformation T: V→W the range of T is a subspace of _____ CO6-U

(a) V (b) W (c) both V and W (d) none of these

9. In a vector space V , if $\langle x, y\rangle = \langle y, z\rangle$ then _____ CO6-U

(a) y = z (b) $y \neq z$ (c) y = - z (d) none of these

10. In a vector space ,$\|ax\| =$ _____ CO6-U

(a) $|a| + \|x\|$ (b) $|a| - \|x\|$ (c) $|a|\|x\|$ (d) $|a|/\|x\|$

PART – B (5 x 2= 10Marks)

11. Evaluate $\int_1^2 \frac{dx}{1+x^2}$ with 2 equal intervals using trapezoidal rule CO1- App

12. Using Euler's method find y(0.2) given $\frac{dy}{dx} = y + e^x$,y(0) =0 CO2- App

13. Write down the Standard Five Point formula and Diagonal Five Point formula to find the numerical solution of the Laplace equation $U_{xx} + U_{yy} = 0$ CO6-U

14. Verify the vectors (1,0,0),(1,1,0) in $R^3$ is a subspace of $R^3$ CO4-App

15. Find the norm of(1,2,3) in $V_3(R)$with standard inner product. CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Compute $f'(0) and f''(4)$ from the data CO1-App (8)

| X: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Y: | 11 | 2.718 | 7.381 | 20.086 | 54.598 |

(ii) Evaluate $\int_0^{\pi} \sin x \, dx$ with 10 equal intervals by CO1-App (8)

(a) Trapezoidal rule

(b) Simpson's $\frac{1}{3}$ rule.

Or

(b) (i) Evaluate $\int_0^2 \frac{dx}{4 + x^2}$ using Romberg's method correct to 4 decimal places. CO1-App (8)

(ii) Evaluate. $\int_1^5 \frac{dx}{x}$ using three point Gaussian quadrature formula. CO1-App (8)

17. (a) (i) Using Taylor's series method find y(0.1)given $y' = x^2 + y^2$ with y(0)=1 CO2-App (8)

(ii) Using R-K method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ CO2-App (8)

with y(0)=1at x=0.2

Or

(b) (i) Given $\frac{dy}{dx}$ =1+ $y^2$, y(0) =0 , y(0.2) = 0.2027 , y(0.4) =0.4228, CO2-App (8)

y(0.6) = 0.6841 evaluate y( 0.8 ) by Adams – Bash forth Method.

(ii) Given $\frac{dy}{dx} = x^3 + y$,y(0)=2,y(0.2)=2.443,y(0.4)=2.99,y(0.6)=3.68, CO2-App (8)

Find y(0.8) by Milne's Predictor & Corrector method

18. (a) (i) Solve $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ , u(0,t) = 0 , u(4,t) = 0 , u(x,0) = x(4 - x) . Take h = 1 and find the values of u up to t = 5 using Bender-Schmidt's difference equation. CO3-App (8)

(ii) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ in $0 \le x \le 5$ , $t \ge 0$ ,u(0,t) = 0 , u(5,t) = 100 , u(x,0) = 20 find the values of u for 1 time step function with h = 1 by Crank-Nicholson's difference method. CO3-App (8)

Or

(b) Solve $\nabla^2 u = -10(x^2+y^2+10)$ over the square mesh with sides x=0,x=3,y=0,y=3 with u=0 on the boundary with h=1. CO3-App (16)

19. (a) (i) Verify the vectors (2,1,0) , (-3,-3,1) ,(-2,1,-1) in $R^3$ is a basis of $R^3$ CO4-App (8)

(ii) Find the dimension of the subspace spanned by the vectors (1,0,2), (2,0,1), (1,0,1) in $V_3(R)$ CO4-App (8)

Or

(b) (i) If T: $R^2 \rightarrow R^2$ be linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ then find nullity(T), rank(T) ,Is T one-to-one? CO4-App (8)

Is T onto? Also check the rank nullity theorem.

(ii) Find the matrix of the linear transformation T: $R^2 \rightarrow R^2$ defined by CO4-App (8)

T(a,b) =(2a-3b,a+b) for the standard basis of $R^2$

| | | | | | |
|---|---|---|---|---|---|
| 20. | (a) | (i) Show that the following function defines an inner product on $V_2(R)$where $x = (x_1, x_2)$ and y= $(y_1, y_2)$ and $\langle x, y\rangle = 6\, x_1 y_1 + 7x_2 y_2$ | CO5-App | (8) |
| | | (ii) If $x = (2,1+i,i)$ and $y = (2-i,2,1+2i)$ then verify Schwarz's inequality. | CO5-App | (8) |
| | | Or | | |
| | (b) | Apply Gram-Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where$v_1 = (1,0,1)$ ,$v_2 = (1,3,1)$ *and* $v_3 = (3,2,1)$ | CO5-App | (16) |