Reg. No: | | | | | | | | | | | | |

Question Paper Code:U3M23

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Third Semester

Electronics and Communication Engineering

21UMA323-Numerical Analysis and Linear Algebra

(Regulations2021)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10Marks)

1. Trapezoidal rule is so called, because it approximates the integral by the sum of _______trapezoids — CO1-U

   (a) n (b) n+1 (c) n-1 (d) 2n

2. Gaussian three point quadrature formula is exact for polynomials up to degree ______ — CO1- U

   (a) 1 (b) 2 (c) 3 (d) 5

3. Taylor Series method will be very useful to give some _____ values for RK, Milne's and Adam's methods — CO2- U

   (a) initial (b) final (c) intermediate (d) two

4. _____ prior values are required to predict the next value in Milne's method — CO2- U

   (a) 1 (b) 2 (c) 3 (d) 4

5. PDE of second order, if $B^2 - 4AC < 0$ then — CO6- U

   (a) parabolic (b) elliptic (c) hyperbolic (d) Non homogeneous

6. Bender-Schmidt recurrence equation is valid if λ= — CO6- U

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

7. The trivial subspaces of a vector space V are _________ — CO4- U

   (a) {0} (b) V (c) W (d) {0}and V

8. The dim(R^3) is ________ CO6- U

(a) 1 (b) 2 (c) 3 (d) 0

9. The norm of $(3,-4,0)$ is _________ CO5- App

(a) 3 (b) -4 (c) 0 (d) 5

10. For any two vectors x and y in an inner product space V, $|\langle x, y\rangle| \leq$ _________ CO6- U

(a) $\|x\|+\|y\|$ (b) $\|x\|\|y\|$ (c) $\|x\|-\|y\|$ (d) $\|x\|/\|y\|$

PART – B (5 x 2= 10Marks)

11. State Newton's backward interpolation formula to compute first two derivatives of y at $x = x_n$ CO1- U

12. Using Taylor's series method find y(0.1) given $y' = 1 + y$ with y(0) = 1 CO2- App

13. State Bender-Schmidt explicit formula to solve the one dimensional heat equation $\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}$ CO3- U

14. State Rank nullity theorem CO4- U

15. Find the norm of $(1,2,3)$ in $V_3(R)$ with standard inner product. CO5 App

PART – C (5 x 16= 80Marks)

16. (a) (i) Calculate the first and second derivatives of y at x = 1 from the following data CO1-App (8)

x	1	2	3	4
y	1	8	27	64

(ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ with 6 equal intervals by CO1-App (8)

(a) Trapezoidal rule

(b) Simpson's $\frac{1}{3}$ rule.

Or

(b) (i) Evaluate $\int_0^2 \frac{dx}{4+x^2}$ using Romberg's method correct to 4 decimal places. CO1 -App (8)

(ii) Evaluate $\int_1^5 \frac{1}{x}dx$ using three point Gaussian quadrature formula CO1 -App (8)

17.	(a)	(i) Using Taylor's series method find y(1.1) given $y' = x + y$ with y(1)=0	CO2 -App	(8)
		(ii) Using Euler's method find y(0.1) and y(0.2) from $y' = 1 - y$, y(0)=0	CO2 -App	(8)
		Or		
	(b)	(i) Using R-K method of fourth order, find y(0.1) for the initial value problem $\frac{dy}{dx} = x + y^2$, y(0) = 1	CO2 -App	(8)
		(ii) Given $\frac{dy}{dx} = x^3 + y$, y(0) = 2 , y(0.2) = 2.443 , y(0.4) = 2.99 , y(0.6) = 3.68 . Calculate y(0.8) by Milne's Predictor & Corrector method.	CO2 -App	(8)
18.	(a)	(i) Solve $\frac{\partial^2 u}{\partial x^2} = 32\frac{\partial u}{\partial t}$, u(0,t) = 0 , u(1,t) = t , u(x,0) = 0 . Take h = 0.25 and find the values of u up to t = 5 using Bender-Schmidt's difference equation.	CO3- App	(8)
		(ii) Using Crank-Nicholson's difference equation to solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ u(0,t) = 0 , u(1,t) = t , u(x,0) = 0. compute u for one time step function with h=0.25 .	CO3- App	(8)
		Or		
	(b)	Solve the Poisson equation $u_{xx} + u_{yy} = -81xy$, 0<x<1, 0<y<1 , u(0,y)=0, u(x,0) = 0, u(1,y)=100, u(x,1)=100 and h=1/3	CO3- App	(16)
19.	(a)	(i) Verify the vectors (1,2,0) , (2,3,0) ,(8,13,0) in R^3 is a basis of R^3	CO4-App	(8)
		(ii) If $T : R^2 \to R^2$ be a linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ then find nullity(**T**) ,rank(**T**), Is **T** one-to-one? Is **T** onto? Also check the rank nullity theorem..	CO4-App	(8)

Or

(b) (i) Find the dimension of the subspace spanned by the vectors (1,2,-3) ,(0,0,1) ,(-1,2,1) in $\boldsymbol{V}_3(\boldsymbol{R})$ — CO4-App (8)

(ii) Find the matrix of the linear transformation $\boldsymbol{T}:\boldsymbol{V}_2(\boldsymbol{R}) \to \boldsymbol{V}_3(\boldsymbol{R})$ defined by $\boldsymbol{T}(\boldsymbol{a},\boldsymbol{b}) = (\boldsymbol{a}+3\boldsymbol{b}, 0, 2\boldsymbol{a}-4\boldsymbol{b})$ for the standard basis of $\boldsymbol{V}_2(\boldsymbol{R})$ — CO4-App (8)

20. (a) Apply Gram-Schmidt process to construct an orthonormal basis for $\boldsymbol{V}_3(\boldsymbol{R})$ with the standard inner product for the basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ where $\boldsymbol{v}_1 = (1,-1,0)$, $\boldsymbol{v}_2 = (2,-1,-2)$ and $\boldsymbol{v}_3 = (1,-1,2)$ — CO5- App (16)

Or

(b) (i) Show that the following function defines an inner product on $\boldsymbol{V}_2(\boldsymbol{R})$ where $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2)$ and $\boldsymbol{y} = (\boldsymbol{y}_1, \boldsymbol{y}_2)$ and $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x}_1\boldsymbol{y}_1 + 2\boldsymbol{x}_2\boldsymbol{y}_2$ — CO5- App (8)

(ii) If $\boldsymbol{x} = (2, 1+\boldsymbol{i}, \boldsymbol{i})$ and $\boldsymbol{y} = (2-\boldsymbol{i}, 2, 1+\boldsymbol{i})$ then verify Schwarz's inequality — CO5- App (8)