Reg. No. : | | | | | | | | | | | | |

Question Paper Code: R2M04

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Electronics and communication Engineering

R21UMA204- CALCULUS,COMPLEX ANALYSIS AND NUMERICAL METHODS

(Regulations R2021)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{1}{(D-m)^2}e^{mx}$ = _____ CO6-U

(a) xe^{mx} (b) x^2e^{mx} (c) $\frac{x^2}{2}e^{mx}$ (d) $\frac{x^2}{m}e^{mx}$

2. $\frac{1}{D^2}(\cos x)$ = _____ CO6- U

(a) sin x (b) - cos x (c) cos x (d) tan x

3. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 2, 1) is _____ CO6- App

(a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 4\vec{j} - \vec{k}$ (c) 3 (d) 9

4. If we take z = logx and $\theta = \frac{d}{dx}$, then $x^2\frac{d^2y}{dx}$ = _____ CO6- U

(a) (θ-1)y (b) θ (θ-1)y (c) $\theta^2 y$ (d) $(\theta^2 - 1)y$

5. The critical point of the transformation $w = z + \frac{1}{z}$ are _____ . CO6- App

a) ± 1 b) ± 2 c) $\pm i$ d) $-i$

6. The constant modulus of the analytic function is CO6- U

(a) constant (b) zero (c) not analytic (d) variable

7. If f(z) is analytic at all points inside and on a simple closed curve c, then $\int_C f(z)dz$ = ------ CO6- U

(a) 2πi (b) -2πi (c) 4πi (d) 0

8. The value of $\int_c \frac{dz}{z+2}$, c : $|z| = 1$ is _____ CO6- U

(a) 2πi (b) -2πi (c) 4πi (d) 0

9. For any root the order of convergence of Newton's method is _____ CO6- U

(a) 4 (b) 1 (c) 2 (d) 3

10. Gauss Seidel iteration converges if the coefficient matrix is _____ dominant CO6- U

(a) Squarely (b) logically (c) diagonally (d) symmetrically

PART – B (5 x 2= 10 Marks)

11. Find Particular integral for $(D^2 - 2D + 1)y = \cosh x$. CO1-App

12. If $\vec{F} = 3\vec{i} + x\vec{j} + y\vec{k}$ show that $curl curl \vec{F} = 0$ CO2-App

13. Show that the function $e^x \sin y$ is harmonic CO3-App

14. Find the Residues of $f(z) = \frac{z+1}{z(z-2)}$ CO4-App

15. What do you mean by diagonally dominant? CO5-U

PART – C (5 x 16= 80 Marks)

16. (a) (i) Solve: $(D^2 - 4D + 3)y = \cos 2x + e^{2x}$ CO1- App (8)

(ii) Using method of variation of parameters solve $(D^2 + a^2)y =$ cosec ax CO1- App (8)

Or

(b) (i) A colony of bacteria is growing exponentially. At time t=0 it has 10 bacteria in it, and at time t=4 it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)

(ii) Solve: $[(2x+1)^2 D^2 - 6(2x+1)D + 16]y = 4(2x+1)^2$ CO1- App (8)

17. (a) Verify Gauss Divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ where S is the surface of the cuboid formed by the planes $x = 0, x = a,$ $y = 0, y = b, z = 0$ & $z = c$. CO2- App (16)

Or

(b) Verify Green's theorem for $\int_C x^2 dx + xy dy$, whereC is bounded by CO2- App (16)

$x = 0, x = a, y = 0, y = a$

18. (a) (i) If f(z) = u +iv is a regular function of z in a domain D the following relation hold in D. $\nabla^2|f(z)|^2 = 4|f'(z)|^2$ CO3- App (8)

(ii) Find the image of $|z-1| = 1$ under the transformation $w = \frac{1}{z}$ CO3- App (8)

Or

(b) (i) Find thebilinear transformation from -i,0,i to -1,i,1. CO3- App (8)

(ii) Find the analytic functions f(z) = u +iv given that CO3- App (8)

$2u + v = e^x(\cos y - \sin y)$

19. Using Contour integration, to prove CO4- App (16)

(a) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{a+b} \quad a > b > 0$

Or

(i) Evaluate $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for the region $1 < |z| < 3$. CO4- App (8)

(b) (ii) Evaluate using Cauchy's Residue theorem for CO4- App (8)

f(z) = $\int_C \frac{e^z dz}{(z^2+\pi^2)^2}$, where 'C' is $|z| = 4$.

20. (a) (i) Solve for a positive root of 3x – cos x - 1 =0 by Newton's Raphson method . CO5- App (8)

(ii) Using Power method find numerically largest Eigen value of CO5- App (8)

$$\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

Or

(b) (i) (i) Solve the equation x^3 –2x-5=0 by iteration method. CO5- App (8)

(ii) (i)Solve 4x + 2y +z = 14, x + 5y -z = 10, x + y + 8z =20 by Gauss Seidel method CO5- App (8)