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**Reg. No. :**

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**Question Paper Code: U2M04**

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Electronics and communication Engineering

21UMA204- Calculus, Complex analysis and Numerical methods

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- $\frac{1}{(D-m)^2} e^{mx} = \underline{\hspace{2cm}}$  CO6-U  
(a)  $x e^{mx}$  (b)  $x^2 e^{mx}$  (c)  $\frac{x^2}{2} e^{mx}$  (d)  $\frac{x^2}{m} e^{mx}$
- The complementary function of  $(4D^2 - 3D - 1)y = 2 \sin 2x$  is  $\underline{\hspace{2cm}}$  CO6-U  
(a)  $Ae^x + Be^{\frac{x}{4}}$  (b)  $Ae^{-x} + Be^{5x}$  (c)  $(A+Bx)e^{2x}$  (d)  $Ae^x + Be^{4x}$
- $\text{Div } \bar{r} = \underline{\hspace{2cm}}$  CO2-App  
(a) 0 (b) 1 (c) 3 (d)  $\bar{r}$
- Divergence of vector  $x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$  at (1, 2, -3) is  $\underline{\hspace{2cm}}$  CO2-App  
(a) 8 (b) 4 (c) -3 (d) 0
- The critical point of the transformation  $w = z + \frac{1}{z}$  are  $\underline{\hspace{2cm}}$  CO3- App  
(a)  $\pm 1$  (b)  $\pm 2$  (c)  $\pm i$  (d)  $-i$
- The function  $f(z) = \frac{1}{z^2+4}$  is not analytic at  $z = \underline{\hspace{2cm}}$  CO3- App  
(a) 2 (b) -2 (c) 2i (d)  $\pm 2i$

7. If  $f(z)$  is analytic at all points inside and on a simple closed curve  $c$ , then  $\int_c f(z)dz = \text{-----}$  CO6-U
- (a)  $2\pi i$                       (b)  $-2\pi i$                       (c)  $4\pi i$                       (d)  $0$
8. Simple pole is a pole of order \_\_\_\_\_ CO6-U
- (a)  $1$                       (d)  $2$                       (c)  $3$                       (d)  $4$
9. Iteration method converges if  $|g^1(x)|$  \_\_\_\_\_ CO6-U
- (a)  $>1$                       (b)  $<1$                       (c)  $=0$                       (d)  $>0$
10. Order of convergence of iteration method is CO5-U
- (a)  $1$                       (d)  $2$                       (c)  $3$                       (d)  $0$

PART – B (5 x 2= 10Marks)

11. Find Particular integral for  $(D^2 - 2D + 1)y = \cosh x$ . CO1-App
12. Find  $\nabla\phi$ , if  $\phi = x^2 + y^2 + z^2$  at  $(1, -1, 1)$ . CO2-App
13. Find the fixed point of  $w = \frac{2z-5}{z+4}$  CO3-App
14. Evaluate  $\int_c \frac{z}{z-2} dz$  where  $C$  is  $|z|=2$  CO4-App
15. State Newton's Iterative formula CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Using method of variation of parameters solve  $(D^2 + a^2)y = \sec ax$  CO1-App (8)
- (ii) At the start of an experiment, there are 100 bacteria. If the bacteria follow an exponential growth pattern with rate  $k = 0.02$ . What will be the population after 5 hours? How long will it take for the population to double? CO1- App (8)
- Or
- (b) (i) Solve:  $(x^2D^2 - xD + 4)y = x^2 \sin(\log x)$  CO1- App (8)
- (ii) Solve:  $(D^2 - 4D + 3)y = \cos 2x + e^{2x}$  CO1- App (8)
17. (a) Verify Gauss Divergence theorem for  $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$  where  $S$  is the surface of the cuboid formed by the planes  $x = 0, x = a,$   
 $y = 0, y = b, z = 0$  &  $z = c$ . CO2-App (16)

Or

- (b) Verify Green's theorem for  $\int_C x^2 dx + xy dy$ , where  $C$  is bounded by  $x = 0$ ,  $x = a$ ,  $y = 0$ ,  $y = a$  CO2 -App (16)

18. (a) (i) Find the image of  $|z - 3i| = 3$  under the transformation  $w = \frac{1}{z}$  CO3-App (8)

- (ii) If  $f(z) = u + iv$  is a regular function of  $z$  in a domain  $D$  the following relation hold in  $D$ .  $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$ . CO3-App (8)

Or

- (b) (i) Find the bilinear transformation from  $-i, 0, i$  to  $-1, i, 1$ . CO3-App (8)

- (ii) Find the analytic functions  $f(z) = u + iv$  given that  $2u + v = e^x (\cos y - \sin y)$  CO3-App (8)

19. (a) (i) Evaluate using Cauchy's Residue theorem for CO4-App (8)

$$f(z) = \int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz, \text{ where 'C' is the circle } |z| = 2.$$

- (ii) Evaluate  $f(z) = \frac{1}{(z+1)(z+3)}$  in Laurent series valid for the region  $1 < |z| < 3$ . CO4-App (8)

Or

- (b) Using contour integration, to find the value of  $\int_0^{2\pi} \frac{d\theta}{13 - 5\cos\theta}$  CO4-App (16)

20. (a) (i) Solve for a positive root of  $3x - \cos x - 1 = 0$  by Newton's Raphson method. CO5-App (8)

- (ii) Solve  $4x + 2y + z = 14$ ,  $x + 5y - z = 10$ ,  $x + y + 8z = 20$  by Gauss Elimination method CO5- App (8)

Or

- (b) (i) Using Power method find numerically largest Eigen value of CO5- App (8)

$$\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

- (ii) Solve by using convolution theorem Solve  $28x + 4y - z = 32$ ;  $x + 3y + 10z = 24$ ;  $2x + 17y + 4z = 35$  by Gauss - Seidel method CO5- App (8)

