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Question Paper Code: U4M21

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Fourth Semester

Computer Science and Engineering

21UMA421 - TRANSFORMS AND DISCRETE MATHEMATICS

(Common to Information Technology & CSE(AIML))

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer All Questions

PART A - (10 x 1 = 10 Marks)

- $\neg P \rightarrow Q$ is equivalent to? CO1- U
(a) $\neg P \wedge Q$ (b) $P \wedge \neg Q$ (c) $\neg(P \wedge Q)$ (d) $P \vee Q$
- P: Rahim is Rich, Q: Rahim is happy. Then 'Rahim is poor but happy' Is best represented by? CO1- U
(a) $\neg P \vee \neg Q$ (b) $P \wedge \neg Q$ (c) $P \wedge Q$ (d) $\neg P \wedge Q$
- $8^n - 3^n$ is divisible by CO2- U
(a) 8 (b) 3 (c) 24 (d) 5
- The numbers between 1 and 100, including both, are divisible by 2 or 6 is _____ CO2- App
(a) 50 (b) 16 (c) 66 (d) 34
- A group $(M, *)$ is said to be abelian if _____ CO3- U
(a) $(x+y)=(y+x)$ (b) $(x*y)=(y*x)$ (c) $(x+y)=x$ (d) $(y*x)=(x+y)$
- The union of two subgroup of G is a CO6- U
(a) Subgroup (b) Semigroup (c) group (d) Monoid
- Fourier transform of $\sqrt{2\pi}$, $-1 < x < 1$ CO4- App
(a) $\frac{\cos s}{s}$ (b) $\sqrt{2\pi} \frac{\cos s}{s}$ (c) $\sqrt{2\pi} \frac{\sin s}{s}$ (d) $\frac{2 \sin s}{s}$

8. A function is called self-reciprocal under Fourier transform, if CO6- U
- (a) It is reciprocal to itself (b) Its Fourier transform is the same function
- (c) Its Fourier transform is its reciprocal (d) None of the above

9. The Z^{-1} transform of $\frac{z}{(z-4)^2}$ is CO5 - U

- (a) 4^n (b) $(-4)^n$ (c) $n(-4)^{n-1}$ (d) $n(4)^{n-1}$

10. The Z -transform of $n3^n$ CO6 - U

- (a) $\frac{z}{(z-3)^2}$ (b) $\frac{z}{(z+3)^2}$ (c) $\frac{3z}{(z-3)^2}$ (d) $\frac{3z}{(z+3)^2}$

PART – B (5 x 2= 10 Marks)

11. Write Demorgan's law? CO1 App
12. In how many ways can letters of the word "MATHEMATICS" be arranged? CO2-App
13. Define Abelian Group and give an example CO3-U
14. Find the Fourier cosine transform of CO2-App
- $e^{-ax}, a > 0$
15. State convolution property of Z – transform. CO5-U

PART – C (5 x 16= 80 Marks)

16. (a) (i) Prove the following by Indirect method. CO1-App (8)
- $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$
- (ii) Using the rules of inference derive CO1-App (8)
- $P \rightarrow Q, (\neg Q \vee R), \neg R, \neg(\neg P \wedge S) \Rightarrow \neg S$

Or

- (b) (i) Prove the following by Indirect Method. CO1-App (8)
- $P \rightarrow Q, Q \rightarrow R, \neg(P \wedge R), P \vee R \Rightarrow R$
- (ii) Prove the following by Indirect method. CO1-App (8)
- $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$

17. (a) (i) Using mathematical induction prove that $(a^n - b^n)$ is divisible by $(a - b)$. CO2- App (8)
- (ii) Solve: $a_{n+2} + 3a_{n+1} + 2a_n = 3^n, a_0 = 0, a_1 = 1$ CO2- App (8)

Or

- (b) (i) Using generating functions Solve: CO2- App (8)
 $a_n - 7a_{n-1} + 10a_{n-2} = 0, a_0 = 10, a_1 = 41.$

- (ii) Calculate the number of positive integers not exceeding 250 CO2- App (8)
that are divisible by 2, 3, 5 or by 7.

18. (a) $S = Q \times Q$, such that binary operation defined by CO3- U (16)
 $(a, b) * (x, y) = (ax, ay + b)$

(i) Prove that $(S, *)$ is a semi group

(ii) Is it commutative?

(iii) Find the identity element of S ?

(iv) Find Inverse element of S ?

Or

- (b) (i) The necessary and sufficient condition for a non-empty subset H CO3- U (8)
of a group $(G, *)$ to be a subgroup is $a, b \in H \Rightarrow a * b^{-1} \in H.$

- (ii) The binary operation $*$ is defined on Q^+ such that CO3- U (8)
 $a * b = \frac{ab}{5}, a, b \in Q^+, \text{ Show that } (Q^+, *) \text{ is an abelian Group.}$

19. (a) Compute the Fourier Transform of CO4- App (16)

$$f(x) = \begin{cases} a - |x| & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases} \text{ and hence evaluate}$$

$$(i) \int_0^{\infty} \left(\frac{\sin x}{x} \right)^4 dx \quad (ii) \int_0^{\infty} \left(\frac{\sin x}{x} \right)^2 dx$$

Or

- (b) Evaluate CO4- App (16)

$$(i) \int_0^{\infty} \frac{dx}{(x^2 + 16)(x^2 + 36)} \quad (ii) \int_0^{\infty} \frac{x^2 dx}{(x^2 + 49)^2} \quad \text{using Fourier transform}$$

20. (a) (i) Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ given that CO5- App (8)
 $y_0 = 0, y_1 = 1$

- (ii) Find the Z transform of $\left(\frac{1}{(n+1)(n+2)} \right)$ CO5- App (8)

Or

- (b) (i) Solve the difference equation $y_{n+2} - 7y_{n+1} + 6y_n = 3^n$ given that $y_0 = 0, y_1 = 0$ CO5- App (8)
- (ii) Find the Z transform of $(a^n \cos n\theta)$ and $(a^n \sin n\theta)$ CO5- App (8)