A   Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐☐

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Question Paper Code: R2M10

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Computer Science and Design

R21UMA210- DIFFERENTIAL EQUATIONS, FOURIER SERIES & TRANSFORMS

(Regulations R2021)

(Common to Artificial Intelligence and Data Science)

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Duration: Three hours   Maximum: 100 Marks

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Answer All Questions

PART A - (10 x 1 = 10 Marks)

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1. The complementary function of $(4D^2 - 3D - 1)y = 2 \sin 2x$ is _____. 　　　　　CO1-App

   (a)$Ae^x + Be^{-\frac{x}{4}}$ 　　(b)$Ae^{-x} + Be^{5x}$ 　　(c) $(A+Bx)e^{2x}$ 　　(d)$Ae^x + Be^{4x}$

2. $\frac{1}{(D-m)^2} e^{mx}$ = _____ . 　　　　　　　　　　　　　　　　　　　CO1-App

   (a) $) xe^{mx}$ 　　(b)$x^2 e^{mx}$ 　　(a) $) xe^{mx}$ 　　(b)$x^2 e^{mx}$

3. L (cosh at) = _____ 　　　　　　　　　　　　　　　　　　　　　　　CO2- App

   (a) $\frac{s}{s^2 - a^2}$ 　　(b)$\frac{a}{s^2 - a^2}$ 　　(c)$\frac{s}{s^2 + a^2}$ 　　(d) $\frac{a}{s^2 + a^2}$

4. L[f(at)] = _____ 　　　　　　　　　　　　　　　　　　　　　　　CO2-App

   (a) $F(\frac{s}{a})$ 　　(b) $aF(\frac{s}{a})$ 　　(c) $\frac{1}{a}F(\frac{s}{a})$ 　　(d) a F(s)

5. Z [n f(n)] = _____ 　　　　　　　　　　　　　　　　　　　　　　CO3- App

   (a) $-z\frac{d}{dz}F(Z)$ 　　(b) $z\frac{d}{dz}F(Z)$ 　　(c) $\frac{d}{dz}F(Z)$ 　　(d) $z^n\frac{d}{dz}F(Z)$

6. Z[a^nf(n)] = _____ 　　　　　　　　　　　　　　　　　　　　　CO6- U

   (a) $F(\frac{z}{a})$ 　　(b) a^n F(z) 　　(c) a^{-n} F(z) 　　(d) F(az)

7. The term ($a_1\cos x + b_1 \sin x$) in Fourier series is called -----------.   CO6 - U

   (a) First harmonic   (b) Second harmonic   (c) Third harmonic   (d) Fourier Coefficients

8. The root mean square value of $f(x) = x^2$ in (0, 1) is ---------.   CO4- App

   (a) 1   (b)½   (c)$1/\sqrt{3}$ (d) 2

9. Convolution theorem on Fourier Transform is F[f(x)*g(x)]=   CO6 – U

   _____

   (a) F(s).G(s)   (b) f(s).g(s)   (c) F(s)*G(s)   (d) f(s)*g(s)

10. If $F[f(x)] = f(s)$ then the function is said to be _____   CO6 - U

    (a) Odd   (b) Even   (c) Self Reciprocal   (d) Periodic

<center>PART – B (5 x 2= 10Marks)</center>

11. Find Particular integral for   CO1 -App

    $(\mathbf{D^2 \text{-} 2D+1})\mathbf{y} = \mathbf{coshx}$.

12. Find $L(te^{2t})$ CO2- App

13. Evaluate   CO3 App

    $$Z\left(\frac{1}{n+1}\right)$$

14. Find b_n in the Fourier series of $f(x) = |\cos x|$ in $(0,2\pi)$.   CO4- App

15. Find the Fourier sine transform of $\frac{1}{x}$ CO5- App

<center>PART – C (5 x 16= 80Marks)</center>

16. (a) (i) Solve $(\mathbf{D^2 \text{-} 3D+2})\mathbf{y} = \mathbf{2e^x + 2cos2x}$ CO1 App   (8)

    (ii) Solve $\left(3z-4y\right)p + \left(4x-2z\right)q = 2y-3x$ CO1 App   (8)

<center>Or</center>

    (b) (i) Solve   CO1 App   (8)

    $(x^2D^2 + 4xD + 2)y = x + \dfrac{1}{x}$

    (ii) Solve $\left(D^2 - D - 6\right)y = 3e^{4x} + 5$ CO1 App   (8)

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17. (a) (i) Find the Laplace transform of   CO2- App   (8)

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \dfrac{\pi}{\omega} \\ 0, & \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases} \quad \text{and} \quad f\left(t + \dfrac{2\pi}{w}\right) = f(t)$$

ii) Solve by using convolution theorem $L^{-1}\left[\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ CO2- App   (8)

Or

(b) i) Solve the differential equation   CO2- App   (8)

$\dfrac{d^2 y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin t$ if

y(0)= $y'(0)$=0 by using Laplace transform method.

ii) Find the Laplace Transforms of $te^{-2t} \sin t$ CO2- App   (8)

18. (a) (i) Evaluate   CO3- App   (8)

$$Z^{-1}\left[\dfrac{z}{z^2 + 7z + 10}\right]$$

(ii) Solve the difference equation $y_{n+2} - 7y_{n+1} + 6y_n = 4^n$ given   CO3- App   (8)
that $y_0 = 0, \ y_1 = 0$

Or

(b) (i) Evaluate $Z[a^n \cos n\theta] \ and \ Z[a^n \sin n\theta]$ CO3- App   (8)

(ii) Using convolution theorem find the inverse Z-   CO3- App   (8)
transform $Z^{-1}\left(\dfrac{8z^2}{(2z-1)(4z-1)}\right)$

19. (a) Find the Fourier series of f(x) = x^2 in $0 < x < 2\pi$ CO4- App   (16)

Or

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(b) i) The table of values of the function $y = f(x)$ is given below: CO4- App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x.

(ii) Find the half range cosine series for $f(x) = \pi - x^2$ in $(0, \pi)$ CO4- App (8)

20. (a) Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & \text{if} \quad |x| \le a \\ 0 & \text{if} \quad |x| > a \end{cases}$ and CO5- App (16)

hence deduce that i) $\int\limits_0^\infty \left(\dfrac{\sin t}{t}\right)^2 dt = \dfrac{\pi}{2}$ ii) $\int\limits_0^\infty \left(\dfrac{\sin t}{t}\right)^4 dt = \dfrac{\pi}{3}$

Or

(b) (i) Find Fourier Cosine and Sine Transform of e^{-ax} hence evaluate CO5- App (8)

$\int\limits_0^\infty \dfrac{dx}{\left(x^2 + 9\right)^2}$

(ii) Evaluate $\int\limits_0^\infty \dfrac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ CO5- App (8)

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