A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M10

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Computer Science and Design

21UMA210- Differential equations, Fourier series & Transforms

(Regulations 2021)

(Common to Artificial Intelligence and Data Science)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The auxiliary equation of the equation $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ is ____. CO1-App

(a) m^2- 4 m +5=0 (b) m^2+3 m -2=0 (c) m^2+3m +2=0 (d) 2 m^2+5m -7=0

2. $\frac{1}{D^2}(\cos x)$ = ____ CO6-U

(a) sin x (b) - cos x (c) cos x (d) tan x

3. L (sin h at) = _____ CO2-App

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

4. L(te^{-3t}) CO2-App

(a) $-z\frac{d}{dz}F(Z)$ (b) $z\frac{d}{dz}F(Z)$ (c) $\frac{d}{dz}F(Z)$ (d) $z^n\frac{d}{dz}F(Z)$

5. Z [n f(n)] = _____ CO3- App

(a) $-z\frac{d}{dz}F(Z)$ (b) $z\frac{d}{dz}F(Z)$ (c) $\frac{d}{dz}F(Z)$ (d) $z^n\frac{d}{dz}F(Z)$

6. The Z transform of $n2^n$ is ____ CO3- App

(a) $\frac{2z}{(z-2)^2}$ (b) $\frac{z}{(z-2)^2}$ (c) $\frac{2z}{(z+2)^2}$ (d) $\frac{z}{(z+2)^2}$

7. The root mean square value of $f(x) = x^2$ in (0, l) is ---------. CO4-App

(a) l (b)½ (c)$1/\sqrt{3}$ (d) 2

8. The constant term in the Fourier expansion for f(x) = k, (0, 2**π**) is CO4-App

(a) 2k (b) 3k (c) k (d) 0

9. Convolution theorem on Fourier Transform is F[f(x)*g(x)]= CO6-U

_____

(a) F(s).G(s) (b) f(s).g(s) (c) F(s)*G(s) (d) f(s)*g(s)

10. If f(x) is an even function then $\int_{-a}^{a} f(x)dx =$____ CO6-U

(a) 0 (b)$\frac{1}{2}\int_0^a f(x)dx$ (c) $2\int_0^a f(x)dx$ (d) $\int_0^a f(x)dx$

PART – B (5 x 2= 10Marks)

11. Find Particular integral for CO1-App

$\mathbf{(D^2 - 2D + 1)y = \cosh x}$.

12. Evaluate $\mathbf{L}[\mathbf{\cos at}]$ CO2-App

13. Evaluate $\mathbf{Z}\left(\frac{\mathbf{1}}{\mathbf{n+1}}\right)$ CO3-App

14. Give the expression for the Fourier series coefficient b_n for the function f(x) = x^2 defined in $-2 \le x \le 2$. CO4-App

15. Find the Fourier Sine Transform of $f(x) = e^{-x}$,x>0 CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve $\mathbf{(D^2 - 3D + 2)y = 2e^x + 2\cos 2x}$ CO1-App (8)

(ii) Solve $(D^2 - D - 6)y = 3e^{4x} + 5$ CO1- App (8)

Or

(b) (i) Solve $\mathbf{(D^2 - 3D + 2)y = 2e^x + 2\cos 2x}$ CO1- App (8)

(ii) Solve $(3z - 4y)p + (4x - 2z)q = 2y - 3x$ CO1- App (8)

17. (a) Find the Laplace transform of $\mathbf{f(t)} = \begin{cases} \mathbf{t}, & \mathbf{0 < t < a} \\ \mathbf{2a - t}, & \mathbf{a < t < 2a} \end{cases}$ and $f(t + 2a) = f(t)$ CO2-App (16)

Or

(b) Solve by the convolution theorem CO2 -App (16)

$$L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$$

18. (a) Using convolution theorem find the inverse Z-transform CO3-App (16)

$$Z^{-1}\left(\frac{8z^2}{(2z-1)(4z-1)}\right)$$

Or

(b) (i) Evaluate $Z[a^n \cos n\theta]$ *and* $Z[a^n \sin n\theta]$ CO3-App (8)

(ii) Evaluate CO3-App (8)

$$Z^{-1}\left[\frac{z}{z^2+4z+3}\right]$$

19. (a) Find the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ of periodicity 2π. CO4-App (16)

Or

(b) (i) The table of values of the function $y = f(x)$ is given below: CO4-App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x.

(ii) Find the half range sine series for $\mathbf{f(x) = x^2}$ in $(\mathbf{0}, l)$ CO4-App (8)

20. (a) Find the Fourier transform of $\mathbf{f(x)} = \begin{cases} 1-|\mathbf{x}|, & \mathbf{if} \quad |\mathbf{x}| \le \mathbf{1} \\ \mathbf{0} & \mathbf{if} \quad |\mathbf{x}| > \mathbf{1} \end{cases}$ and CO5-App (16)

hence deduce that *i)* $\int_0^\infty \left(\frac{\mathbf{sint}}{\mathbf{t}}\right)^2 \mathbf{dt} = \frac{\pi}{2}$ ii) $\int_0^\infty \left(\frac{\mathbf{sint}}{\mathbf{t}}\right)^4 \mathbf{dt} = \frac{\pi}{3}$

Or

(b) (i) Find the Fourier sine & cosine transform of e^{-ax} CO5-App (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(x^2+25)^2}$ CO5-App (8)