Reg. No: | | | | | | | | | | | | |

Question Paper Code: U4M26

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Fourth Semester

Computer Science and Design

21UMA426- PROBABILITY AND STATISTICAL TECHNIQUES

(Common to Artificial Intelligence and Data Science Engineering)

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10 x 1 = 10 Marks)

1. A Continuous r.v has a p.d.f $f(x)=\frac{1}{4}\ 0\le x\le 4$, then value of $E(X^3)$ CO1- App

(a) 16 (b) 64 (c) 256 (d) 512

2. The mean of the random variable is denoted by CO6- U

(a) E(X) (b) $E(X^2)$ (c) 0 (d) 1

3. Which of the following distribution has equal mean and Variance? CO6 -U

a) Geometric (b)Poisson (c) Normal (d) Binomial

4. If $M_x(t)=(0.3+0.7e^t)^{10}$ then value of mean is CO6 -U

(a) 30 (b) 0 .21 (c) 70 (d) 21

5. For a set of five bivariate data(x, y) covariance is 10. Variance of x and y are 25 and 16 respectively. The Karl Pearson coefficient of correlation is CO3 – App

(a) 1 (b) -1 (c) 1/2 (d) 1/4

6. The joint probability density function is f(x, y) = k, 0 < x < 2, 0 < y < 1. Estimate K = CO3 – App

a) 4 b) -1/2 c) 1/2 d) 5

7. Estimate and estimator are: CO 6 – U

(a) Same (b) Different (c)Maximum (d) Minimum

8. If T is an unbiased estimator for θ, then T^2 is a …….. Estimator for θ^2. CO 6 – U

(a) unbiased (b) biased (c) Both (a) & (b) (d) None of the above

9. The mean for t-test distribution is CO6- U

(a) $t = \frac{\bar{x}_1 - \mu}{s/\sqrt{n-1}}$ (b) $t = \frac{\bar{x}_1 + \mu}{s/\sqrt{n-1}}$ (c) t = 0 (d) None of the above

10. F-test is used to test for equality of _____ CO6- U

(a) Mean (b) Variance (c) ratio (d) all the above

PART – B (5 x 2= 10 Marks)

11. A coin is tossed twice; find the probability that there will appear atleast one tail? CO1 -App

12. If Moment generating function $M_X(t) = \frac{5}{5-t}$, find the variance value CO2- App

13. Define Regression lines and also find the angle between these two lines? CO3 -U

14. Distinguish between point and interval estimate. CO6 -U

15. State the applications of Chi-square distribution CO6 -U

PART – C (5 x 16= 80 Marks)

16. (a) (i) The joint probability mass function of (X,Y) is given by $P(x,y) = k(x+3y)$ $x = 0, 1, 4$ $y = 1, 2, 3$ Compute marginal distribution function , and conditional distribution CO1 – App (8)

(ii) A RV X has the following distribution Compute Mean and Variance CO1 – App (8)

x	0	1	2	3	4	5	6	7	8
P(X	a	3a	5a	7a	9a	11a	13a	15a	17a

Or

(b) (i) The cumulative distribution function of a random variable X is $F(x) = 1-(1+x)e^{-x}, x > 0$. Find the probability density function of X, mean and variance. CO1 – App (8)

(ii) If X and Y are two random variables with joint pdf $f(x,y) = K(6-x-y),\ 0 < x < 2,\ 2 < y < 4$ CO1 – App (8)

Compute (a) K (b) Marginal density function of x (c) E (X)

17. (a) (i) Compute the moment generating function of Poisson distribution and hence Compute it's mean and variance. CO2 –App (8)

(ii) If X is a normal variate with mean 30 and variance 25. Compute the probabilities that Compute CO2 –App (8)

$P(25 \le X \le 40)$, $P(X \ge 36)$, $P(|X - 34| > 3)$

Or

(b) (i)The number of typing mistakes that a typist makes on a given page has a Poisson distribution with a mean of 3 mistakes. What is the probability that she makes (i) Exactly 7 mistakes (ii) fewer than 4 mistakes and (iii) no mistakes on a given page? CO2 –App (8)

(ii) Compute the moment generating function of Binomial distribution and hence Compute it's mean and variance CO2 –App (8)

18. (a) (i) Obtain the rank Correlation coefficient for the following data: CO3 App (8)

X	12	15	17	18	12	16	15	27
Y	14	10	14	13	16	10	14	15

(ii) Obtain the Correlation coefficient for the following heights (in inches) of fathers X and their sons Y. CO3- App (8)

X	58	56	59	57	58	50	60	64
Y	67	68	65	68	72	72	69	75

Or

(b) (i) Joint pdf of X and Y is CO3 -App (8)

$$f(x,y) = \begin{cases} x+y & ,0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & elsewhere \end{cases}$$

Compute Regression Equations

(ii) If $\sigma_1 = 2,\ \sigma_2 = \sigma_3 = 3,\ r_{12} = 0.7,\ r_{23} = r_{31} = 0.6$ Compute CO3- App (8)

(i)$r_{23.1}$ (ii)$R_{1.23}$ (iii)$b_{13.2}$ (iv)$b_{12.3}$

19. (a) (i) A sample of size n is drawn from each of the four normal populations which have the same variance σ^2. The means of the four populations are CO4 - App (8)

a + b + c, a + b – c, a – b + c and a – b – c. What are the MLE's for a, b, c and σ^2.

(ii) In random sampling from normal population N(μ, σ^2), find the maximum likelihood estimators for CO4- App (8)

(i) μ when σ^2 is known

(ii) σ^2 when μ is known and

(iii) The simultaneous estimation of μ and σ^2.

Or

(b) (i) A manufacturer of ball pens claims that a certain pen he manufactures has a mean writing life of 400 pages with a standard deviation of 20 pages. A purchasing agent selects a sample of 100 pens and puts them for test. The mean writing life for the sample was 390 pages. Should the purchasing agent reject the manufactures claim at 1% level? CO4 - App (8)

(ii) Show that $\frac{\sum x_i \left[\sum x_i - 1\right]}{n(n-1)}$ is an unbiased estimate of θ^2, for the sample $x_1, x_2, x_3 \cdots x_n$ drawn on X which takes the values 1 or 0 with respective probabilities θ and (1 – θ). CO4 – App (8)

20. (a) (i) A certain injection administered to each of 12 patients resulted in the following increases of blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 can it be classified that the injection will be, in general, accompanied by an increase in BP? CO5 – Ana (8)

(ii) The table gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week. CO5 – Ana (8)

Days	Mon	Tue	Wed	Thu	Fri
No. of accidents	8	12	9	14	17

Or

(b) (i) Two researchers adopted different sampling techniques while investigating the same group of students to find the number of students falling into different intelligence level. The results are as follows CO5 – Ana (8)

Research	Below Average	Average	Above Average	Excellent
X	40	33	25	2
Y	86	60	44	10

(ii) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. CO5 – Ana (8)

Sample I	17	13	12	15	12	14	16	14	15
Sample II	16	19	13	16	18	13	15	--	--

Do the estimates of the population variance differ significally at 5% level?