A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M08

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Computer Science and Business Systems

21UMA208- LINEAR ALGEBRA AND NUMERICAL METHODS

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If the Eigen value of a matrix A are 1,2,3 then the Eigen value of A^T CO1-App

   (a)2,4,6 (b) 1,4,9 (c) 2,8,18 (d)1,2,3

2. If $A=\begin{pmatrix} a & 1 \\ 3 & b \end{pmatrix}$ has Eigen values of 2,-2 then a and b are_______ CO1-App

   (a)1,-1 (b) -1,-1 (c)1,1 (d)0,1

3. Solve the linear system 5x+4y=15,3x+7y=12 gauss –Jordan method CO2-App

   (a) $\frac{57}{23},\frac{15}{23}$ (b) $\frac{15}{23},\frac{15}{23}$ (c) $\frac{5}{23},\frac{15}{23}$ (d) $\frac{57}{23},\frac{5}{23}$

4. By Gauss elimination method, solve $x+y=2$, $2x+3y=5$ CO2-App

   (a)1,2 (b) 1,1 (c) 1,0 (d) 0,1

5. Gauss Seidel method iteration converges if the coefficient matrix is _______dominant CO3- U

   (a) Squarely (b) Logically (c) Diagonally (d) Symmetrically

6. The order of convergence of Newton's method is _______ CO3- U

   (a) 1 (b) 2 (c) 3 (d) 0

7. In a vector space V, for every $x, y \in V$ then property x+y=y+x is known as________ CO6- U

   (a) Commutative (b) Associative (c) identity (d) Inverse

8.	The dim(R^3) is _______		CO6- U
	(a) 1	(b) 2	(c) 3	(d) 0

8. The dim(R^3) is _______ — CO6- U

(a) 1 (b) 2 (c) 3 (d) 0

9. In a vector space find $\|\alpha x\| =$ _____ — CO6- U

(a) $|\alpha| + \|x\|$ (b) $|\alpha| - \|x\|$ (c) $|\alpha|\|x\|$ (d) $|\alpha| / \|x\|$

10. The norm of $(3,-4,0)$ is _________ — CO6- U

(a) 3 (b) -4 (c) 0 (d) 5

PART – B (5 x 2= 10 Marks)

11. Construct the matrix of the quadratic forms $2x_1x_2 + 2x_2x_3 - 2x_3x_1$ — CO1-App

12. Apply Gauss –Jordan method solve the linear system $x + y = 2; 2x + 3y = 5$. — CO2-App

13. Explain Newton's backward interpolation formula — CO6- U

14. Find the matrix of $T : V_2(R) \to V_3(R)$ given by $T(a,b) = (a+3b, 0, 2a-4b)$ for the standard Basis of $V_2(R)$ — CO4-App

15. Explain inner product space — CO6-U

PART – C (5 x 16= 80Marks)

16. (a) Using Cayley Hamilton theorem find A^4 and A^{-1} when — CO1-App (16)

$$A=\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$

Or

(b) Apply the orthogonal transformation reduce the following quadratic forms into canonical form $Q = 6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$, find its rank, index, signature and nature — CO1-App (16)

17. (a) (i) Apply Gauss elimination method to solve 2x+y+4z=12,8x-3y+2z=1,4x+11y-z=33 — CO2-App (8)

(ii) Apply Gauss Jordan method to solve 10x+y+z=12, 2x+10y+z=13, x+y+5z=7 — CO2-App (8)

Or

(b) Solve the following using triangularisation method x+y+z=9,2x-3y+4z=13,3x+4y+5z=40 CO2-App (16)

18. (a) (i) Using Lagrange's interpolation formula calculate the profit in the year 2000 from the following data : CO3-App (8)

year	1997	1999	2001	2002
Profit (Rs.in lakhs)	43	65	159	248

(ii) Apply Newton Raphson Method Calculate a root of $x\log_{10} x - 1.2 = 0$ correct to 3 decimals. CO3-App (8)

Or

(b) Calculate the dominant Eigen value and corresponding Eigen vector of A. if CO3-App (16)

A= $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

19. (a) Construct the linear transformation $T:V_3(R) \to V_3(R)$ determine by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ with respect the standard basis of $V_3(R)$ CO4-App (16)

Or

(b) Let $T: R^2 \to R^3$ be the linear mapping defined by $T(a_1 + a_2, a_1 - a_2, a_2)$, Calculate nullity(T),rank(T), Also check the rank nullity theorem CO4-App (16)

20. (a) Apply Gram-Schmidth process to construct an orthonormal basis for $V_3(R)$ with standard inner product for the basis $\{V_1, V_2, V_3\}$where $V_1 = (1,0,1), V_2 = (1,0,-1)$ $and V_3 = (0,3,4)$. CO5-App (16)

Or

(b) Show that $V_2(R)$ is an inner product space with inner product defined by $\langle x, y \rangle = x_1 y_1 + x_2 y_1 + x_1 y_2 + 4x_2 y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$ CO5-App (16)