A   Reg. No. :   [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

First Semester

R21UMA102- MATRIX & CALCULUS

(Common to ALL branches except CSBS)

(Regulations R2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The Index of the matrix A is the number of _____ Eigen values of A   CO6- U

   (a) zero   (b) non zero   (c) negative   (d) positive

2. Nature of the Canonical form $f(x_1, x_2,) = 2x_1^2 + x_2^2$ CO6- U

   (a) positive definite   (b) negative definite   (c) positive semi definite   (d)indefinite

3. The $(n+1)^{th}$ derivative of x^n CO6- U

   (a) $n!$ (b) $(n+1)!$ (c) n (d) 0

4. The nth derivative of x^{2n} CO6- U

   (a) $(2n)!$ (b) $2n$ (c) $n!$ (d) None

5. The stationary points of $x^2 - xy + y^2 - 2x + y$ is _____   CO3 - App

   (a) (-1, 0)   (b) 0, -1   (c) (1, 0)   (d) (0, 1)

6. If AC-B^2>0 and A<0, then f(a,b) has a _____ value.   CO6 - U

   (a) minimum   (b)maximum   (c)not extremum   (d) inconclusive

7. $\int_{0}^{\infty} e^{-x} x^4 dx =$ CO4 - App

   (a) 4   (b)4!   (c) 5   (d) 5!

8. Value of $\Gamma(6)$ is equal to   CO4 - App

   (a) 5!   (b) 4   (c) 5   (d) 6

9. $\int_0^1 \int_0^2 \int_0^3 dxdydz$ is equal to   CO5 - App

   (a) 2   (b) 3   (c) 4   (d) 6

10. The region of integration of the integral $\int_0^1 \int_0^x f(x,y)\,dxdy$ is   CO6 - U

   (a) square   (b) rectangle   (c) triangle   (d) circle

<div align="center">PART – B (5 x 2= 10Marks)</div>

11. Find the Eigen values of A^{-1} and $A^2 + 3I$ for the matrix   CO1 - App

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

12. Evaluate $\displaystyle\lim_{x \to 1} \left(\frac{4x^3 - 1}{x - 1} \right)$ CO2- App

13. If u = F (x-y, y-z, z-x), prove that   CO3 -App

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

14. Find the value of   CO4 - App

$$\int_0^\infty x^7 e^{-x}\,dx$$

15. Sketch the region of integration   CO5 - App

$$\int_0^1 \int_0^x f(x,y)\,dy\,dx$$

<div align="center">PART – C (5 x 16= 80Marks)</div>

16. (a)   (i) Find the Eigen values and Eigen Vectors of   CO1 - App   (8)

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

   (ii) Using Cayley-Hamilton theorem find   CO1 - App   (8)

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<div align="right">**R1M02**</div>

$$A^{-1} \quad \text{for} \quad A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 5 & 4 \\ -1 & 3 & 2 \end{bmatrix}$$

Or

(b) Reduce the Q.F $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to a canonical form by an orthogonal transformation and hence find rank, signature, index and nature   CO1 - App   (16)

17. (a) (i) Suppose that a corpse was discovered in a hotel room at midnight and its temperature was 80°F. The temperature of the room is kept constant at 60°F. Two hours later the temperature of the corpse dropped to 75°F. Find the time of death.   CO2 - Ana   (8)

(ii) Expand $e^{\cos x}$ by Maclaurin's series up to the term containing x^4 CO2 - App   (8)

Or

(b) (i) The radioactive Isotope Indium-III is often used for diagonosis and imaging in nuclear medicine. Its half-life is 2.8days. What was the initial mass of the isotope before decay, if the mass in 2 weeks was 5g.   CO2 -Ana   (8)

(ii) Find the nth derivative of   CO2 - App   (8)

$$\frac{x}{2x^2 + 3x + 1}$$

18. (a) (i) Find the dimensions of rectangular box without top of maximum capacity with surface area 432 square meters   CO3 -App   (10)

(ii) If $u = a \cosh x \cos y$, $v = a \sinh x \sin y$ .then show that   CO3 -App   (6)

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{2}a^2(\cosh 2x - \cos 2y)$$

Or

(b) (i) Using Taylor's series expand $e^x \sin y$ about $\left(1, \dfrac{\pi}{2}\right)$ up to third degree terms.   CO3 -App   (8)

(ii) Obtain the extreme values of the function $f(x,y) = x^3 y^2 (1 - x - y)$ CO3 -App   (8)

R1M02

19. (a) (i) Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ CO4 -App   (10)

(ii) Compute $\displaystyle\int_{0}^{\frac{\pi}{2}} \dfrac{1}{1+\cot x}\,dx$ CO4 -App   (6)

Or

(b) (i) Determine the reduction formula for $\int \cos^{n} x\,dx$ CO4 -App   (8)

(ii) Prove that $\beta\left(m,\dfrac{1}{2}\right) = 2^{2m-1}\,\beta(m,m)$ CO4 -App   (8)

20. (a) Using the Triple integration, compute the volume of the Sphere   CO5 -App   (16)

$x^{2}+y^{2}+z^{2} = a^{2}$

Or

(b) (i) Changing into polar coordinates and hence evaluate   CO5 -App   (8)

$\displaystyle\int_{0}^{2}\int_{0}^{\sqrt{2x-x^{2}}} \left(x^{2}+y^{2}\right)\,dy\,dx$

(ii) Using the double integration, compute the area of the circle   CO5 -App   (8)

$x^{2}+y^{2} = a^{2}$

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R1M02