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Question Paper Code: U1M02

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

First Semester

Civil Engineering

21UMA102- MATRIX AND CALCULUS

(Common to ALL branches)

(Regulations 2021)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- The sum and product of the Eigen values of $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are _____ CO1- App
(a) 4, -4 (b) 0, -4 (c) 4, 2 (d) 5, 3
- If 0, 3, 4 are the Eigen values of a matrix A then $|A| =$ _____ CO6- U
(a) 0 (b) 1 (c) 2 (d) 3
- If $y = \cos(ax+b)$ then nth derivative $y_n =$ _____ CO2- App
(a) $a^n \cos\left(ax - b - \frac{n\pi}{2}\right)$ (b) $a^n \sin(ax+b)$ (c) $a^n \cos\left(ax+b + \frac{n\pi}{2}\right)$ (d) $\cos(ax+b)$
- If $E = 90$, $R = 12$, $L = 3$, then Kirchoff's Law is _____ CO2- App
(a) $\frac{di}{dt} + 4i = 30$ (b) $3\frac{di}{dt} + 90i = 12$ (c) $4\frac{di}{dt} + i = 30$ (d) $30\frac{di}{dt} + 4i = 1$
- If $x = r \cos \theta$, $y = r \sin \theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} =$ _____ CO3 - App
(a) θ (b) r (c) $\frac{1}{r}$ (d) $\frac{1}{\theta}$

6. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then $\cos u$ is a homogeneous function of degree ____ CO3- App
- (a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 0
7. $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx$ is ____ CO4- App
- (a) $\frac{\pi}{85}$ (b) $\frac{3}{256}$ (c) $\frac{\pi}{32}$ (d) $\frac{5\pi}{32}$
8. $\int_0^{\infty} e^{-x} x^4 \, dx =$ CO4 - App
- (a) 4 (b) 4! (c) 5 (d) 5!
9. The region of integration of the integral $\int_0^1 \int_0^x f(x, y) \, dx \, dy$ is CO5 - App
- (a) square (b) rectangle (c) triangle (d) circle
10. $\int_0^1 \int_0^2 \int_0^3 dx \, dy \, dz$ is equal to CO5 - App
- (a) 2 (b) 3 (c) 4 (d) 6

PART – B (5 x 2= 10Marks)

11. Find the Eigen values of A^{-1} and $A^2 + 3I$ for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ CO1-App
12. Evaluate $\lim_{x \rightarrow 0} \frac{6^x - 3^x}{x}$ CO2- App
13. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ CO3- App
14. Compute $\Gamma\left(\frac{7}{2}\right)$ CO4- App
15. Evaluate $\int_0^1 \int_0^3 (x+y) \, dy \, dx$ CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Find the Eigen values and Eigen Vectors of $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ CO1- App (8)

(ii) Using Cayley-Hamilton theorem find CO1-App (8)

$$A^4 \text{ for } A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

Or

- (b) Reduce the Q.F $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to a canonical form by an orthogonal transformation and hence find rank, signature, index and nature CO1- App (16)

17. (a) (i) A pot of boiling water 100°C is removed from the fire and allowed to cool at 30°C room temperature. Two minutes later, the temperature of the water in the pot is 90°C . What will be the temperature of the water after 5 minutes? CO2- Ana (8)

(ii) Expand $e^{\sin x}$ by Maclaurin's series up to the term containing x^4 CO2- App (8)

Or

- (b) (i) If $y = e^{ax} \cos bx$, prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$ CO2- App (8)

(ii) Find the n^{th} derivative of $\frac{x^2 + x - 1}{x^3 + x^2 - 6x}$ CO2- App (8)

18. (a) (i) A rectangular box open at the top is to have a volume of 32cc, calculate the dimensions of the box that requires least material for its constructions CO3- App (10)

(ii) If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ then show that $\frac{\partial(\mathbf{u}, \mathbf{v}, \mathbf{w})}{\partial(\mathbf{x}, \mathbf{y}, \mathbf{z})} = 4$ CO3- App (6)

Or

- (b) (i) Using Taylor's series expand $e^x \sin y$ near the point $\left(-1, \frac{\pi}{4}\right)$ up to third degree terms. CO3- App (8)

(ii) Obtain the extreme values of the function CO3- App (8)

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

19. (a) (i) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ CO4- App (10)

(ii) Compute $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{(\cos x) + \sqrt{(\sin x)}}} dx$ CO4- App (6)

Or

(b) (i) Determine the reduction formula for $\int \cos^n x dx$ CO4- App (10)

(ii) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m, m)$ CO4- App (6)

20. (a) Using the Triple integration, compute the volume of the Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ CO5- App (16)

Or

(b) (i) Show that the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3} a^2$ CO5- App (8)

(ii) Change the order of integration and hence evaluate $\int_0^a \int_0^a (x^2 + y^2) dy dx$ CO5- App (8)