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**Question Paper Code :R3M25**

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Third Semester

Civil Engineering

R21UMA325-PROBABILITY, STATISTICS AND TRANSFORM TECHNIQUES

(Regulations R2021)

Duration: Three hours

Maximum: 100 Marks

PART A - (10 x 1 = 10 Marks)

- Probability of an impossible event is CO6-U  
(a) 1 (b) 10 (c) 0 (d) 100
- Which of the following continuous distribution has equal mean and variance? CO6-U  
(a) Exponential (b) Binomial (c) Gamma (d) Normal
- The degrees of freedom in t-tests is CO6-U  
(a) n-1 (b) n-2 (c) n-3 (d) n-4
- F-test is used to test for equality of \_\_\_\_\_ CO6-U  
(a) Mean (b) Variance (c) ratio (d) all the above
- $\cos x$  is a periodic function with period ----- CO6-U  
(a)  $\pi$  (b)  $2\pi$  (c)  $\pi/3$  (d)  $2\pi/3$
- The Fourier constant  $b_n$  in  $(-\pi, \pi)$  for  $x \sin x$  is \_\_\_\_\_ CO3-App  
(a)  $x^2$  (b)  $3x$  (c) 0 (d) 1
- If  $F[f(x)] = f(s)$  then the function is said to be \_\_\_\_\_ CO6-U  
(a) Odd (b) Even (c) Self Reciprocal (d) Periodic
- $F_s[e^{-ax}] =$  \_\_\_\_\_ CO6-U  
(a)  $\sqrt{\frac{2}{\pi}} \frac{s}{s^2 + a^2}$  (b)  $\sqrt{\frac{2}{\pi}} \frac{a}{s^2 + a^2}$  (c)  $\sqrt{\frac{2}{\pi}} \frac{a^2}{s^2 + a^2}$  (d)  $\sqrt{\frac{2}{\pi}} \frac{s^2}{s^2 + a^2}$

9. The Z transform of  $n2^n$  is \_\_\_\_\_ CO5-App
- (a)  $\frac{2z}{(z-2)^2}$  (b)  $\frac{z}{(z-2)^2}$  (c)  $\frac{2z}{(z+2)^2}$  (d)  $\frac{z}{(z+2)^2}$

10.  $Z\left\{\frac{1}{n+1}\right\}$  CO5-U
- (a)  $z \log \left[ \frac{z}{z-1} \right]$  (b)  $\frac{z}{(z-2)^2}$  (c)  $\frac{2z}{(z+2)^2}$  (d)  $\frac{z}{(z+2)^2}$

PART – B (5 x 2= 10Marks)

11. Find the mean for the discrete RV X with probability distribution CO1-App

|      |     |     |     |     |
|------|-----|-----|-----|-----|
| X    | -2  | -1  | 0   | 1   |
| P(X) | 0.4 | 0.1 | 0.2 | 0.3 |

12. Explain Null Hypothesis CO2-U
13. Find  $a_0$  and  $a_n$  in the Fourier series of  $f(x) = x$  in  $(0, 2\pi)$ . CO3-App
14. Find the Fourier sine transform of  $\frac{1}{x}$  CO4-App
15. State Initial and final value Theorem on Z Transform CO6-U

PART – C (5 x 16= 80Marks)

16. (a) (i) If the density function of a continuous random variable X is given by CO1-App (8)

$$f(x) = \begin{cases} ax & ; 0 \leq x \leq 1 \\ a & ; 1 \leq x \leq 2 \\ 3a - ax & ; 2 \leq x \leq 3 \\ 0 & otherwise \end{cases}$$

(a) Find the value of “a” (b). Find

the distribution function of X

- (ii) Define Poisson distribution. Find the moment generating function and hence find mean and variance CO1-App (8)

Or

- (b) (i) Compute the moment generating function of Binomial distribution and hence Compute its mean and variance CO1-App (8)
- (ii) The number of monthly breakdowns of a computer is a R.V. having a Poisson distribution with mean equal to 1.8. Find the Probability that his computer will function for a month CO1-App (8)
- (a) Without a breakdown (b) With only one breakdown (c) With at least one breakdown

17. (a) (i) The following data are collected on two characters.

CO2-Ana (8)

|             | Smokers | Non Smokers |
|-------------|---------|-------------|
| Literates   | 83      | 57          |
| Illiterates | 45      | 68          |

Using chi-square test to find is there any relation between smoking and literacy.

(ii) Five coins are tossed 256 times. The number of heads observed is given below. Examine if the coins are unbiased, by employing  $\chi^2$  goodness of fit.

CO2-Ana (8)

| No of Heads | 0 | 1  | 2  | 3  | 4  | 5  |
|-------------|---|----|----|----|----|----|
| Frequency   | 5 | 35 | 75 | 84 | 45 | 12 |

Or

(b) (i) From the following information state identify the condition of the child is associated with the condition of the house.

CO2-Ana (8)

| Condition of Child | Condition of House |       | Total |
|--------------------|--------------------|-------|-------|
|                    | Clean              | Dirty |       |
| Clean              | 69                 | 51    | 120   |
| Fairly Clean       | 81                 | 20    | 101   |
| Dirty              | 35                 | 44    | 79    |
| Total              | 185                | 115   | 300   |

(ii) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. Identify the sampling distribution. Do the estimate of the population variance differ significantly.

CO2-Ana (8)

| Sample I  | 17 | 13 | 12 | 15 | 12 | 14 | 16 | 14 | 15 |
|-----------|----|----|----|----|----|----|----|----|----|
| Sample II | 16 | 19 | 13 | 16 | 18 | 13 | 15 |    |    |

18. (a) Express  $f(x) = \frac{1}{2}(\pi - x)$  as a Fourier series of period  $2\pi$  in the interval  $0 < x < 2\pi$ .

CO3-App (16)

Or

- (b) The table of values of the function  $y = f(x)$  is given below: CO3-App (16)

|    |     |         |          |       |          |          |        |
|----|-----|---------|----------|-------|----------|----------|--------|
| x  | 0   | $\pi/3$ | $2\pi/3$ | $\pi$ | $4\pi/3$ | $5\pi/3$ | $2\pi$ |
| y: | 1.0 | 1.4     | 1.9      | 1.7   | 1.5      | 1.2      | 1.0    |

Find a Fourier series upto the third harmonic for  $f(x)$  in terms of  $x$ .

19. (a) Compute the Fourier Transform of  $f(x) = \begin{cases} a - |x| & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$  CO4-App (16)

and hence evaluate i)  $\int_0^{\infty} \left(\frac{\sin x}{x}\right)^4 dx$  ii)  $\int_0^{\infty} \left(\frac{\sin x}{x}\right)^2 dx$

Or

- (b) (i) Compute  $\int_0^{\infty} \frac{x^2 dx}{(x^2 + 4)(x^2 + 9)}$  using Fourier transform CO4-App (8)

(ii) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + 16)^2}$  using Fourier transform CO4-App (8)

20. (a) (i) Solve the difference equation  $y_{n+2} - 6y_{n+1} + 8y_n = 5^n$  given that  $y_0 = 0, y_1 = 0$  CO5-App (8)

(ii) Using Convolution theorem find  $Z^{-1} \left[ \frac{10z^2}{(5z-2)(2z+1)} \right]$  CO5-App (8)

Or

- (b) (i) Evaluate  $Z[r^n \cos n\theta]$  and  $Z[r^n \sin n\theta]$  CO5-App (8)

(ii) Evaluate  $Z^{-1} \left[ \frac{z^3}{(z-1)^2(z-2)} \right]$  using partial fraction CO5-App (8)