Question Paper Code:U3M25

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Third Semester

CivilEngineering

21UMA325-PROBABILITY, STATISTICS AND TRANSFORM TECHNIQUES

(Regulations2021)

Duration: Three hours   Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10Marks)

1. Probability of sure event is equal to   CO6-U

    (a)0   (b)1   (c) 2   (d) 10

2. The mean of the random variable is denoted by   CO6- U

    (a) E(X)   (b) E(X^2)   (c) 0   (d) 1

3. The degrees of freedom in t-tests is   CO6- U

    (a) n-1   (b) n-2   (c) n-3   (d) n-4

4. Choose the t-test for mean   CO6- U

    (a) $t = \dfrac{\overline{x_1} - \mu}{s/\sqrt{n-1}}$ (b) $t = \dfrac{\overline{x_1} + \mu}{s/\sqrt{n-1}}$ (c) t = 0   (d) None of the above

5. If f(x + t) = f(x), then f(x) is said to be an _____   CO6- U

    (a) Odd Function   (b) Even Function   (c) Periodic function   (d) Self Reciprocal

6. The Fourier constant b_n in (-π,π) for x sin x is _____   CO6- U

    (a) x^2 (b) 3x   (c) 0   (d) 1

7. $F[xf(x)] =$ _____   CO6- U

    (a) $-F_c[f(x)]$ (b)$-\dfrac{d}{ds}\{F_s[f(x)]\}$ (c)$-F_s[f(x)]$ (d) $-\dfrac{d}{ds}\{F_c[f(x)]\}$

8. $F_s[e^{-ax}] =$ _____   CO6- U

    (a) $\sqrt{\dfrac{2}{\pi}}\dfrac{s}{s^2+a^2}$ (b)$\sqrt{\dfrac{2}{\pi}}\dfrac{a}{s^2+a^2}$ (c)$\sqrt{\dfrac{2}{\pi}}\dfrac{a^2}{s^2+a^2}$ (d)$\sqrt{\dfrac{2}{\pi}}\dfrac{s^2}{s^2+a^2}$

9. The Z transform of a unit step function is _____. CO5- U

    (a) $\log(\frac{z}{z+1})$ (b) $\frac{z}{z+1}$ (c) $\log(\frac{z}{z-1})$ (d) $\frac{z}{z-1}$

10. Evaluate $Z(\frac{1}{\mathbf{n!}})$ CO5- U

    (a) $e^{-1/z}$ (b) $e^{1/z}$ (c) e^{2z} (d) $e^{1/z} - 2$

<div align="center">PART – B (5 x 2= 10Marks)</div>

11. I Find the mean for the discrete RV X with probability distribution CO1- App

X	-2	-1	0	1
P(X)	0.4	0.1	0.2	0.3

12. Write the important properties of F-distribution CO2- U

13. Find b_n in the Fourier series of $f(x) = |\cos x|$ in $(0, 2\pi)$. CO3- App

14. State Fourier integral theorem. CO6- U

15. Prove that $Z(\sin\frac{n\pi}{2}) = \frac{z}{z^2+1}$ CO5 App

<div align="center">PART – C (5 x 16= 80Marks)</div>

16. (a) A Random Variable X has the following probability distribution CO1-App (16)

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2+a$

    Find (i) 'a'

        (ii) $P(X < 6)$, $P(X \geq 6)$, $P(0 < X < 4)$

        (iii) $P(X < 6/ X > 4)$

        (iv) Find the minimum value of 'λ' such that $P(X \leq \lambda) > \frac{1}{2}$

<div align="center">Or</div>

    (b) (i) Using the probability mass function for Binomial distribution, Compute the moment generating function and hence find its mean and variance. CO1 -App (8)

        (ii) Using an Exponential distribution State and Prove the memory less property.. CO1 -App (8)

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17. (a) (i) A sample analysis of examination results of 500 students was made. It was found that 220 students have failed, 170 have secured a third class, 90 have secures a second class and the rest, a first class. So these figures support the general belief that the above categories are in the ratio 4:3:2:1 respectively? CO2 -Ana (8)

(ii) Two group of students A and B were tested , the marks obtained were as follows CO2 -Ana (8)

A	18	20	36	50	49	36	34	49	41
B	29	28	26	35	30	44	46		

Examine the significance of difference between the average marks secured by the students of the above two groups

Or

(b) (i) Two horses A and B were tested according to time (in seconds) to run on a particular track with the following results: CO2 -Ana (8)

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether horse A is running faster than B at 5% level.

(ii) A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight. CO2 -Ana (8)

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	8		

Find the variances are significantly different

18. (a) Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO3- App (16)

Or

(b) The table of values of the function $y = f(x)$ is given below: CO3- App (16)

x:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.8	0.3	0.5	2.16	1.3	1.76	1.8

Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x

19. (a) Compute the Fourier Transform of $f(x)=\begin{cases}1-x^2 & |x|<1 \\ 0 & otherwise\end{cases}$ and hence evaluate the value of (i) $\int\limits_{0}^{\infty}\dfrac{\sin t - t\cos t}{t^3}dt$ CO4-App (16)

$\int\limits_{0}^{\infty}\left(\dfrac{\sin t - t\cos t}{t^3}\right)^2 dt$

Or

(b) Compute (i) $\int\limits_{0}^{\infty}\dfrac{dx}{(x^2+1)(x^2+4)}$ (ii) $\int\limits_{0}^{\infty}\dfrac{x^2 dx}{(x^2+9)^2}$ using Fourier transform CO4-App (16)

20. (a) (i) Solve the difference equation $y_{n+2}-6y_{n+1}+8y_n = 5^n$ given that $y_0 = 0$, $y_1 = 0$ CO5- App (8)

(ii) Using Convolution theorem find $Z^{-1}\left[\dfrac{10z^2}{(5z-2)(2z+1)}\right]$ CO5- App (8)

(b) (i) Solve the difference equation $y_{n+2}+3y_{n+1}-10y_n = 3^n$ given that $y_0 = 0$, $y_1 = 0$ CO5- App (8)

(ii) Using Convolution theorem find $Z^{-1}\left[\dfrac{14z^2}{(7z+3)(2z-1)}\right]$ CO5- App (8)