A

Reg. No. :

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Civil Engineering

R21UMA206- DIFFERENTIAL EQUATIONS, COMPLEX ANALYSIS & TRANSFORM TECHNIQUES

(Regulations R2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The order and degree of $(y''')^2 + 2(y'')^3 + y = 0$ is _____   CO6- U

   (a) 3,2   (b) 2,3   (c) 3,3   (d) 2,2

2. The solution of $(D^3 + D^2 - D - 1)y = 0$ is _____   CO1- Apply

   (a) $Ae^x + Bxe^x + Cx^2e^x$ (b) $(Ax + B)e^x + C e^{-x}$ (c) $e^{-x} + (\cos 2x + i \sin 2x)$ (d) $(Ax + B)e^{-x} + C e^x$

3. The Curl value of $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ is ---   CO2- Apply

   (a) 1   (b) 0   (c) 2   (d) 3

4. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at $(1, 1, 1)$ is _____   CO2- Apply

   (a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3   (d) 9

5. The general solution of $2r + 5s - 3t = 0$ is _____   CO3- Apply

   (a) $f_1 (y + 3x) + f_2 (2y - x)$ (b) $f_1 (3y + x) + f_2 (y + 2x)$ (c) $f_1 (y -3x) + f_2 (2y + x)$ (d) $f_1 (3y - x) + f_2 (y - 2x)$

6. The PDE of all planes through the origin is ___.   CO3- Apply

   (a) $z= px+qy^2$ (b) $z= px- qy^2$ (c) $z= px+qy$ (d) $z= px- qy$

7. The residue of $f(z) = \dfrac{4}{z^3(z-2)}$ at its simple pole is _____

    (a) $\dfrac{4}{7}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{7}$ (d) $\dfrac{3}{4}$

8. The order of pole $z = 0$ of the following functions $f(z) = \dfrac{e^z}{z}$

    (a) 1,0   (b) 1,-1   (c) 1,2   (d) 0,0

9. $L\left(\displaystyle\int_0^t e^{-t}dt\right) =$ _____

    (a) $\dfrac{1}{s}$ (b) $\dfrac{1}{(s+1)}$ (c) $\dfrac{1}{s(s+1)}$ (d) $\dfrac{s}{(s+1)}$

10. Sin t is a periodic function with period ____

    (a) 2π (b) π (c) $\pi / 2$ (d) $\pi / 3$

<center>PART – B (5 x 2= 10Marks)</center>

11. Solve:
$$\left(D^3 - 3D^2 + 3D - 1\right)y = 0$$

12. Compute $\nabla (\log r)$

13. Solve the PDE. $p + q = 8$

14. Evaluate $\displaystyle\int_c \dfrac{e^{-z}}{z+1}dz$ where C is $|z| = \dfrac{1}{2}$ using Cauchy integral formula

15. Find $L[t\cos at]$

<center>PART – C (5 x 16= 80Marks)</center>

16. (a) (i) At the start of an experiment, there are 200 bacteria. If the bacteria follow an exponential growth pattern with rate k = 0.05. What will be the population after 8 hours? How long will it take for the population to double?

    (ii) Using method of variation of parameters solve $(D^2 + a^{2)}y = \tan ax$.

<center>Or</center>

  (b) (i) Solve $(D^2 - 3D + 2)y = e^x + \cos 2x$

    (ii) Solve: $(x^2D^2 + xD + 1)y = x \sin (\log x)$

17. (a) Verify Gauss Divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ where S is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0 \,\&\, z = c$.

<div align="center">Or</div>

(b) Verify Green's theorem for $\int_C x^2\,dx + xy\,dy$, where C is bounded by   CO2- App   (16)

$x = 0,\ x = a,\ y = 0,\ y = a$

18. (a) (i) Solve $z = px + qy + pq$ CO3- App   (8)

(ii) Form a PDE by eliminating arbitrary functions from   CO3- App   (8)

$\varphi(x^2 + y^2 + z^2, x + y + z) = 0$

<div align="center">Or</div>

(b) (i) Solve $\left(D^2 - 6DD' + 5D'^2\right)z = e^{x+y} + \sin(2x + y)$ CO3- App   (8)

(ii) Solve $(y - z)p + (z - x)q = (x - y)$.   CO3- App   (8)

19. (a) (i) Evaluate using Cauchy's Residue theorem for   CO4- App   (8)

$f(z) = \int_C \dfrac{z}{(z-1)(z-3)}\,dz$, where 'C' is the circle $|z| = 2$.

(ii) Evaluate $f(z) = \dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for the   CO4- App   (8)

region
$1 < |z| < 3$.

<div align="center">Or</div>

(b) Using contour integration, to compute the value of $\displaystyle\int_0^{2\pi} \dfrac{d\theta}{13 + 5\cos\theta}$ CO4- App   (16)

20. (a) (i) Solve the differential equation   CO5- App   (8)

$\dfrac{d^2 y}{dt^2} - 3\dfrac{dy}{dt} + 2y = e^{-t}$ with $y(0) = 0\ \&\ y'(0) = 0$ by using Laplace
transform method.

(ii) Compute the Laplace Transforms of $te^{-t}\cos 3t$ CO5- App   (8)

<div align="center">Or</div>

(b) i) Solve the differential equation $\dfrac{d^2 y}{dt^2} + 9y = \cos 2t$ if y(0) = 1 &   CO5- App   (8)

$y\left(\dfrac{\pi}{2}\right) = -1$ by using Laplace transform method.

ii) Compute the Laplace Transforms of $te^{-2t}\sin t$ CO5- App   (8)

R2M06

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