A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M06

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Civil Engineering

21UMA206- Differential Equations, Complex Analysis & Transform Techniques

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The complete solution of $(x^2D^2 - 3xD - 5)y = 0$ is ___ CO1-App

   (a) $Ae^{-z} + B\,e^{5z}$ (b) $Ae^{z} + B\,e^{5z}$

   (c) $Ae^{z} + B\,e^{-5z}$ (d) $Ae^{-z} + B\,e^{-5z}$

2. $\frac{1}{D^2}(\cos x)$ = ____ CO1-App

   (a) sin x (b) - cos x (c) cos x (d) tan x

3. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is _____ CO2-App

   (a) $2(\vec{i} + \vec{j} + \vec{k})$ (b) $2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3 (d) 9

4. If φ is a vector point function then Curl (gradφ) = CO2-App

   (a) 1 (b)0 (c)2 (d) None of the above

5. The PDE obtained from z = (x+a)(y+b) is __. CO3- App

   (a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

6. The PDE of all planes having equal intercepts on the X axis and Y axis is _____ CO3- App

   (a) p = q (b) p + q = 0 (c) pq = 1 (d) p(q + 1) = q

7. Find the poles of $f(z) = \frac{z^2 + 1}{1 - z^2}$ CO6-U

   (a)1,0 (b)1,-1 (c)1,2 (d)0,0

8. Find the order of pole z = 0 of the following functions $f(z) = \frac{e^z}{z}$ CO6-U

(a)0 (b)3 (c)2 (d)1

9. L (sin h at) = _____ CO6-R

(a) $\frac{s}{s^2 - a^2}$ (b) $\frac{a}{s^2 - a^2}$ (c) $\frac{s}{s^2 + a^2}$ (d) $\frac{a}{s^2 + a^2}$

10. $L^{-1}(1)$ = _____ CO6-R

(a) t (b) $\frac{1}{t}$ (c) $\frac{1}{s}$ (d) $\delta(t)$

PART – B (5 x 2= 10Marks)

11. Compute the particular integral of $(D^2 + 5D + 6)y = e^{2x}$ CO1-App

12. Compute if $\varphi = x^2 + y^2 + z^2$ at (2, 0, 1). CO2-App

13. Form the PDE of all spheres whose centre lie on Z- axis CO3-App

14. State Cauchy's residue theorem. CO6-U

15. Find $L\left[te^{at}\right]$ CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Using method of variation of parameters solve $(D^2 + 4)y = \sec 2x$. CO1-App (8)

(ii) A colony of bacteria is growing exponentially. At time t=0 it has 10 bacteria in it, and at time t=4 it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)

Or

(b) (i) Solve: $(x^2D^2 + xD + 1)y = x \sin(\log x)$ CO1- App (8)

(ii) Solve: $(D^2 - 4D + 3)y = \sin 3x + e^{2x}$ CO1- App (8)

17. (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped x = 0, x = a, y = 0, y = b, z = 0, z = c. CO2-App (16)

Or

(b) (i) Using Green's theorem, Evaluate $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region defined by CO2 -App (8)

X = 0, Y = 0, X + Y= 1 in the XY plane.

(ii) Prove that $\bar{F} = (x^2 + xy^2)\bar{\imath} + (y^2 + x^2y)\bar{\jmath}$ is irrotational vector and compute the Scalar potential such that $\bar{F} = \nabla\emptyset$. CO2 -App (8)

18. (a) (i) Solve: $(mz - ny)\,p + (nx - lz)\,q = ly - mx$ CO3-App (8)

(ii) Solve $(D^2 - DD' - 3D'^2)z = \sin(x + y) + e^{6x+y}$ CO3-App (8)

Or

(b) (i) Solve $p^2 + q^2 = x^2 + y^2$ CO3-App (8)

(ii) Form a PDE by eliminating arbitrary functions from $z = px + qy + p^2 - q^2$ CO3-App (8)

19. (a) (i) Evaluate $f(z) = \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)}\,dz$ by using Cauchy's Integral formula where C is $|z| = 3$ CO4-App (8)

(ii) Expand $\frac{z-1}{(z+2)(z+3)}$ as Laurent's series valid in the region $2 < |z| < 3$ CO4-App (8)

Or

(b) Using contour integration, to compute the value of $\int_0^{2\pi} \frac{d\theta}{13 + 5\cos\theta}$ CO4-App (16)

20. (a) (i) Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{-t}$ with $y(0) = 0 \,\&\, y'(0) = 0$ by using Laplace transform method. CO5-App (8)

(ii) Compute the Laplace Transforms of $te^{-t}\cos 3t$ CO5-App (8)

Or

(b) (i) Using the periodic function, Find the Laplace transform of f(t) = CO5-App (8)

$$\begin{cases} E, & 0 < t < b \\ -E, & b < t < 2b \end{cases}$$

(ii) Using Convolution Theorem, Compute $L^{-1}\left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$ CO5-App (8)