A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R1M02

B.E./B.Tech. DEGREE EXAMINATION, NOV/DEC 2024

First Semester

Civil Engineering

R21UMA102- MATRIX AND CALCULUS

(Regulations R2021)

(Statistical Tables may be permitted)

(Common to CSE, EEE, ECE,MECH, IT, CHEMICAL, AGRI, BME, BIOTECH, CSD, AI&DS, CSE(AIML), CSE(SC) & CSE(IOT) Engineering Branches)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If the Eigen values of a matrix A are -2,3,2 then the Eigen values of $2A^T$ are CO6- U

(a) 4,-6,-8 (b) -4,6,8 (c) 8,18,32 (d) 8,18,8

2. If two Eigen values of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal to 1 each, then the determinant of A^{-1} is CO6- U

(a) 1 (b) 5 (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

3. $\lim_{\theta \to 0} \frac{\cos\theta}{\theta} =$ ___ CO6- U

(a) 0 (b) 2 (c) 1 (d) does not exist

4. The $(n+1)^{th}$ derivative of x^{n+1} CO6- U

(a) $n!$ (b) $(n+1)!$ (c) n (d) 0

5. $F = f + \lambda g$ where f is called _____ CO3-U

(a) Function of variable (b) Jacobian (c) Lagrangian (d) Constraint

6. If $AC - B^2 < 0$ and $A > 0$, B is positive at (a, b) then the point (a, b) is called — CO3-U

(a) minimum (b) maximum (c) Saddle (d) inconclusive

7. The value of $\Gamma\left(\frac{5}{2}\right)$ — CO4-App

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\sqrt{\pi}}{4}$ (d) $\frac{\sqrt{\pi}}{2}$

8. The value of $\beta(1,6)$ — CO4-App

(a) 1 (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) None

9. The value of $\int \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} \dots\dots\dots\right) dx$ — CO6-U

(a) $-e^{-x}$ (b) e^{x} (c)) e^{2x} (d)) e^{-2x}

10. $\int \frac{cotx}{sinx} dx =$ _____ — CO6-U

(a) $\log(cosecx + cotx)$ (b) $\log(cosecx - cotx)$

(c) $-\log(cosecx + cotx)$ (d) $-cosecx$

PART – B (5 x 2= 10Marks)

11. The product of two Eigen values of — CO1 - App

A=$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 4. Determine the Eigen values of A^{-1}

12. Evaluate — CO2 – App

$$\lim_{x \to 0} \frac{e^{2x}}{x^5}$$

13. Compute the stationary points of the function $5x^2 + y^2 - 4y$ — CO3 – App

14. Compute the value of $\int_0^1 x(1-x)^{15} dx$ — CO4 – App

15. Change the order of integration $\int_0^2 \int_x^2 f(x,y)dxdy$ — CO5 – App

PART – C (5 x 16= 80Marks)

16. (a) (i) Reduce the Quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$ to the canonical form through an orthogonal transformation and hence find rank, signature, index and nature — CO1–App (16)

Or

(b) (i) Using Cayley-Hamilton theorem compute A^{-1} for $A = \begin{bmatrix} -1 & 0 & 3 \\ 8 & 1 & -7 \\ -3 & 0 & 8 \end{bmatrix}$ — CO1 –App (8)

(ii) Compute the Eigen values and Eigen Vectors of $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ — CO1-App (8)

17. (a) (i) A body originally at 80^0 C cools down to 60^0 C in 20 minutes, the temperature of the air being 30^0 C . What will be the temperature of the body after 33 minutes from the original? — CO2 –App (8)

(ii) Find the n^{th} derivative of $\frac{4x-9}{(2x-1)(6x-5)}$ — CO2 –App (8)

Or

(b) (i) The initial mass of an Iodine isotope was 168g. Determine the Iodine mass after 30 days if the half life of the isotope is 12 days? — CO2 –App (8)

(ii) If 30% of radioactive substance disappeared in 10 days, , Examine how long will it take for 60% of it to disappear? — CO2 –App (8)

18. (a) (i) If $u = x + y + z, uv = y + z, uvw = z$ compute the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ — CO3 –App (8)

(ii) Compute the maximum and minimum value of $x^3 + y^3 - 3x - 75y + 120$ — CO3 –App (8)

Or

(b) (i) Compute the dimensions of rectangular box without top of maximum capacity with surface area 432 square meter — CO3 –App (8)

(ii) Expand $x^3 + y^3 + x^2y + y - 4x$ at $(1,3)$ using Taylor's expansion up to third degree terms — CO3 –App (8)

19. (a) (i) Evaluate $\int_0^1 \frac{x}{\sqrt{(1-x^4)}}dx$ CO4 –App (8)

(ii) Evaluate $\int_0^1 \frac{dx}{\sqrt{(1-x^3)}}$ CO4 –App (8)

Or

(b) (i) Compute the value of CO4 –App (8)

$\int_0^{\frac{\pi}{2}} \frac{x}{sinx+cosx}dx$

(ii) Compute the integral CO4 –App (8)

$\int_0^{\infty} \frac{x^c}{c^x}dx$

20. (a) (i) Using the Triple integration, compute the volume of the Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ CO5 –App (8)

(ii) Change the order of integration and hence evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydxdy$ CO5 –App (8)

Or

(b) (i) Compute the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4,\ z=0$ CO5 –App (8)

(ii) Compute $\iiint \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the first octant for the sphere $x^2+y^2+z^2=a^2$ CO5 –App (8)