Reg. No. :

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Artificial Intelligence and Machine Learning

R21UMA211-FOURIER SERIES, PARTIAL DIFFERENTIAL EQUATIONS AND

COMPLEX ANALYSIS

(Regulations R2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If a function f(x) is even, its Fourier expansion contains only ------- terms   CO6- U

   (a) Sine   (b)Cosine   (c)tan   (d) None of these

2. The Fourier constant b_n in $(-\pi,\pi)$ for x sin x is _____   CO1-App

   (a) x^2 (b)3x   (c)0   (d) 1

3. The roots of $(D^2 - 4DD' + 3D'^2)\, z = 0$ is_____.   CO2-App

   (a) (-1 , 3)   (b) (1, -3)   (c) (-1, -3)   (d) (1, 3)

4. The complete solution of $z = px + qy + pq$ is -----   CO2-App

   (a) $z = ax + by + ab$ (b) $z = ax - by - ab$ (c) $z = ax + by - ab$ (d) $z = ax - by + ab$

5. Classify the equation $u_{xx} + u_{yy} = 0$ is _____   CO3-App

   (a) parabolic   (b)hyperbolic   (c) elliptic   (d) cyclic

6. In one dimensional heat equation $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}, \alpha^2$ is the ____ of the substance.   CO3-App

   (a) diffusivity   (b) specific heat   (c) thermal conductivity   (d) density

7. The function $f(z) = \dfrac{1}{z^2+4}$ is not analytic at $z =$ ____   CO6- U

   (a) 2   (b) -2   c) 2i   (d) $\pm 2i$

8. The function $f(z) = \frac{1}{z^2+4}$ is not analytic at $z =$ ____   CO6- U

    (a) 2   (b) -2   c)2i   (d)$\pm$2i

9. The value of $\int_c \frac{dz}{z+2}$, $c : |z| = 1$ is ____   CO6- U

    (a) 2πi   (b) -2πi   (c) 4πi   (d) 0

10. Simple pole is a pole of order ____   CO6- U

    (a) 1   (b) 2   (c) 3   (d) 4

<div align="center">PART – B (5 x 2= 10 Marks)</div>

11. State Dirichlet's conditions   CO1- App

12. Solve $\sqrt{p} + \sqrt{q} = 1$ CO2- App

13. Classify the differential equation $3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - u = 0$ CO3-App

14. Show that the function $e^x \sin y$ is harmonic   CO4- App

15. Find the Residues of $f(z) = \frac{z+1}{z(z-2)}$ CO5- App

<div align="center">PART – C (5 x 16= 80Marks)</div>

16. (a)   (i) Find the Half range cosine series for $f(x) = x$ in $(0, \pi)$.   CO1- App   (8)

          Deduce that $\displaystyle\sum_{n=odd}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{96}$

          (ii) Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2п. Hence deduce that the value of the sum   CO1-App   (8)
$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

<div align="center">Or</div>

(b)   (i) Compute first two harmonics of the Fourier series for the following data.   CO1-App   (8)

x	0	2	4	6	8	10	12
y	10	12	20	24	26	17	10

          (ii) Find the Half range cosine series for $f(x) = 2x$ in $(0, \pi)$ CO1-App   (8)

R2M11

17. (a)　(i) Solve $\left(D^2 - 5DD' + 6D'^2\right)z = e^{x+y} + \sin(x-y)$　CO2-App　(8)

(ii) Solve $x(y-z)p + y(z-x)q = z(x-y)$　CO2-App　(8)

Or

(b)　(i) Solve $Z = px + qy + p^2 - q^2$　CO2-App　(8)

(ii) Form a PDE by eliminating arbitrary functions from $\varphi(x^2 + y^2 + z^2, x + y + z) = 0$.　CO2-App　(8)

18. (a)　A bar of 10cm long with insulated sides has its ends A and B kept at 50° c and 100^oc respectively. Until steady state condition prevails. The temperature at A is then suddenly raised to 90^oc and at the same instant B is lower to 60^oc and maintained thereafter. Find the subsequent temperature distribution in the bar.　CO3 -App　(16)

Or

(b)　If a string of length 'l' is initially at rest in its equilibrium position and each of its points is given velocity, $\dfrac{\partial y}{\partial t} = V_0 \sin^3 \dfrac{\pi x}{l}$,0<x<l, Determine the displacement function y(x,t)　CO3- App　(16)

19. (a)　(i) Using Milne Thomson method, find the Analytic function given that $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$　CO4-App　(8)

(ii) Find the image of $|z-1|=1$ under th transformation $w = \dfrac{1}{z}$　CO4-App　(8)

Or

(b)　(i) Find the bilinear transformation from -1,0,1 to 0,i,3i　CO4-App　(8)

(ii) If f(z) = u +iv is a regular function of z in a domain D the following relation hold in D. $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$　CO4-App　(8)

R2M11

20. (a)　　(i) Evaluate using Cauchy's Integral formula for　　　　　CO5-App　(8)

$$f(z) = \int_C \frac{2z-1}{(z+1)(z-3)} dz, \text{ where 'C' : } |z| = 2.$$

(ii) Find the Laurent's series of $f(z) = \dfrac{7z-2}{z(z+1)(z-2)}$ valid

in the region $1 < |z+1| < 3$

　　　　　　　　　　　　　　　CO5-App　(8)

Or

(b)　　Evaluate: $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{(x^2+4)(x^2+9)} dx$, using contour integration.　　CO5-App　(16)

R2M11