A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M11

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Artificial Intelligence and Machine Learning

21UMA211-FOURIER SERIES, PARTIAL DIFFERENTIAL EQUATIONS AND COMPLEX ANALYSIS

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If a function f(x) is even, its Fourier expansion contains only ------- terms CO6- U

(a) Sine (b) Cosine (c) tan (d) None of these

2. cos x is a periodic function with period ---------- CO1- App

(a) π (b) 2π (c) $\pi/3$ (d) $2\pi/3$

3. The PDE of all planes through the origin is ___. CO2-App

(a) $z= px+qy^2$ (b) $z= px- qy^2$ (c) $z= px+qy$ (d) $z= px- qy$

4. The general solution of $2r + 5s – 3t = 0$ is _____ CO2-App

(a) $f_1 (y + 3x) + f_2 (2y - x)$ (b) $f_1 (3y + x) + f_2 (y + 2x)$

(c) $f_1 (y -3x) + f_2 (2y + x)$ (d) $f_1 (3y - x) + f_2 (y - 2x)$

5. $Au_{xx}+Bu_{xy}+Cu_{yy} = f(x, y)$ is parabolic if_____. CO6- U

(a) $B^2-4AC<0$ (b) $B^2-4AC= 0$ (c) $B^2-4AC>0$ (d) $B^2-4AC\neq0$

6. An insulated rod of length 60 cm has its ends at A and B kept at 20^oC and 80^oC respectively, then its steady state solution is CO3-App

(a) x-20 (b)4x+20 (c) x+20 (d) x+60

7. The critical point of the transformation $w = z + \frac{1}{z}$ are ____ CO4- App

a) ±1 b) ±2 a) ±1 b) ±2

8. Find the fixed points of $f(z)=\frac{1}{z-2i}$ CO4-App

(a) i (b) 2i (c) 3i (d) 0

9. The residue of f(z) = $\frac{4}{z^3(z-2)}$ at its simple pole is _____ CO5-App

(a) $\frac{4}{7}$ (b) $\frac{3}{4}$ (c) $\frac{1}{7}$ (d) $\frac{3}{4}$

10. The value of $\int_C \frac{dz}{z+2}$, c : $|z| = 1$ is _____ CO6-U

(a) 2πi (b) 0 (c) 4πi (d) 0

PART – B (5 x 2= 10Marks)

11. State Dirichlet's conditions CO1–App

12. Form the PDE from $z = ax^n + by^n$. CO2–App

13. Classify $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8u_y - 16u = 0$ CO3-App

14. Find the fixed point of $w=\frac{2z-5}{z+4}$ CO4-App

15. Evaluate $\int_c \frac{z}{z-2}dz$ where C is $|z| = 2$ CO5–App

PART – C (5 x 16= 80Marks)

16. (a) (i) Express $f(x)=(\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. CO1- App (8)

(ii) The table of values of the function $y = f(x)$ is given below: CO1-App (8)

x:	0	T/6	T/3	T/2	2T/3	5T/6	T
y:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Find a Fourier series up to the third harmonic to represent f(x) in terms of x

Or

(b) (i) Find the Half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. CO1- App (8)

Deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}.....=\frac{\pi^4}{90}$

(ii) The table of values of the function $y = f(x)$ is given below: CO1-App (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Find a Fourier series upto the third harmonic to represent f(x) in terms of x

17. (a) (i) Solve $(D^2 - 5DD' + 6D'^2)z = e^{x+y} + sin(x-y)$ CO2 -App (8)

(ii) Solve $x(y-z)p + y(z-x)q = z(x-y)$ CO2 -App (8)

Or

(b) (i) Solve $Z = px + qy + p^2 - q^2$ CO2 -App (8)

(ii) Form a PDE by eliminating arbitrary functions from CO2 -App (8)
$\varphi(x^2 + y^2 + z^2, x + y + z) = 0$..

18. (a) A bar of 30cm long with insulated sides has its ends A and B kept at 20^o c and 80^oc respectively. Until steady state condition prevails. The temperature at A is then suddenly raised to 60^oc and at the same instant B is lower to 40^oc and maintained thereafter. Find the subsequent temperature distribution in the bar. CO3-App (16)

Or

(b) A String is stretched and fastened to two points l apart.Motion is started by displacing the string into the form $y=K(lx-x^2)$ from which it is released at t=0.Find the displacement of any point at a distance 'x' at any time 't' CO3-App (16)

19. (a) (i) Using Milne Thomson method, find the Analytic function CO4-App (8)
given that $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

(ii) Find the bilinear transformation from -1,0,1 to 0,i,3i CO4-App (8)

Or

(b) (i) Find the image of $|z-1| = 1$ under the transformation CO4-App (8)
$w = \dfrac{1}{z}$

(ii) If f (z) is analytic whose real part is constant must itself be a constant CO4-App (8)

20. (a) (i) Evaluate using Cauchy's Residue theorem for CO5-App (8)

f(z) = $\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)}\, dz$, where 'C' is the circle $|z| = 2$.

(ii) Evaluate $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for the region CO5-App (8)

$1 < |z| < 3$.

Or

(b) Using contour integration, to find the value of $\int_0^{2\pi} \frac{d\theta}{13 - 5\cos\theta}$ CO5-App (16)