Reg. No : | | | | | | | | | | | | |

Question Paper Code: U5M01

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Fifth Semester

Artificial Intelligence And Data Science

21UMA501-LINEAR ALGEBRA

(Regulations 2021)

Duration: Three hours — Maximum: 100 Marks

PART A - (10 x 1 = 10 Marks)

1. The Union of two Subspaces of a Vector Space is a Subspace if and only if one is -------------------------in the other. CO6- U

   (a) Need not Contained (b) Contained (c) Dependent (d) None of these

2. Any Vector in the form $\alpha_1 v_1 + \alpha_2 v_2 + \ldots\ldots + \alpha_n v_n$ is called ------- CO6- U

   (a) Linear transformation (b) Linear Combination

   (c) Linear independent (d) none of these

3. If T: V→W be linear transformation then T(0) = _________ CO6- U

   (a) 0 (b) 1 (c) 2 (d) 3

4. Any linear transformation T: V→W can be represented by a _________ CO6- U

   (a) line (b) circle (c) square (d) matrix

5. $\langle x, x \rangle = 0$ if and only if _________ CO6- U

   (a) $x = 1$ (b) $x \neq 1$ (c) $x = 0$ (d) $x \neq 0$

6. In a vector space , $\|\alpha x\| =$ _________ CO6- U

   (a) $|\alpha| + \|x\|$ (b) $|\alpha| - \|x\|$ (c) $|\alpha| \|x\|$ (d) $|\alpha| / \|x\|$

7. All Eigen values of a __________ operator are real CO6- U

(a) Self adjoint (b) Eigen Vector (c) Inner Product (d) None of these

8. Any matrix can be put in to the factorial form PDQ, where P and Q are Unitary and D is __________ CO6- U

(a) Dimension (b) Diagonal (c) Determinant (d) None of these

9. Linear Programming deals with the ------------ of a function of decision variable. CO6- U

(a) Optimization (b) Formulation (c) Technique (d) None of these

10. Find b_n in the expansion of x^2 as a Fourier series in $(-\pi,\pi)$ CO6- U

a) π b) $-\pi$ c) 0 (d) None of these

PART – B (5 x 2= 10Marks)

11. Verify the commutative property for a vector space $R \times R$ over R under addition defined by $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ CO1-App

12. State any two properties of linear transformation CO6-U

13. If $x = (2, 1+i, i)$ and $y = (2-i, 2, 1+2i)$ then find $\langle x, y \rangle$. CO3-App

14. State Cayley – Hamilton theorem CO4-App

15. Write down the usage of Linear algebra in Computer Graphics CO6-U

PART – C (5 x 16= 80Marks)

16. (a) (i) Verify the vectors (2, 1, 0), (-3, -3, 1), (-2, 1, -1) in R^3 is a basis of R^3. CO1- App (8)

(ii) Prove that R X R is a Vector Space over under addition and Scalar multiplication defined by $(x_1, x_2) + (y_1,y_2) = (x_1+y_1, x_2+y_2)$ and $\alpha(x_1,x_2) = (\alpha x_1,\alpha x_2)$. CO1- App (8)

Or

(b) (i) Find the dimension of the subspace spanned by the vectors (2, 0, 1), (-1, 0, 1), (1, 0, 2) in $V_3(R)$. CO1- App (8)

(ii) Write the matrix E= $\begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ as a linear combination of the matrix CO1- App (8)

A = $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, B = $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ and C = $\begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}$

17.	(a)	(i) If T: $R^2 \to R^3$ be linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1 - a_2, a_2)$ then find nullity(T),rank(T), Is T one-to-one? Is T onto? Also check the rank nullity theorem.	CO2- App	(8)
		(ii) Find the matrix of the linear transformation T: $V_3(R) \to V_3(R)$ defined by T(a,b,c) = (a,a+b,2a+b+3c) for the standard basis of $V_3(R)$.	CO2- App	(8)
		Or		
	(b)	Find the Linear transformation T : $R^3 \to R^3$ such that T(1, 1, 1) = (1, 1, 1) T(1, 2, 3) = (-1, -2, 3) and T(1, 1, 2) = (2, 2, 4)	CO2- App	(16)
18.	(a)	Apply Gram-Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1 = (1,0,1)$, $v_2 = (1,3,1)$ and $v_3 = (3,2,1)$	CO3- App	(16)
		Or		
	(b)	If $x = (2, 1+i, i)$ and $y = (2-i, 2, 1+2i)$ then verify Schwarz's inequality.	CO3- App	(16)
19.	(a)	Determine the Cholesky Decomposition for A = $\begin{pmatrix} 16 & -3 & 5 & -8 \\ -3 & 16 & -5 & -8 \\ 5 & -5 & 24 & 0 \\ -8 & -8 & 0 & 21 \end{pmatrix}$	CO4- App	(16)
		Or		
	(b)	Construct a QR factorization for a matrix A = $\begin{pmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{pmatrix}$	CO4- App	(16)

20. (a) An economy consists of the coal, electric (Power) and steal sectors and the output of each sectors is distributed among the various sectors as listed below CO5- App (16)

Distribution from output

Coal	Electric	Steel	Purchased by
0.0	0.4	0.6	Coal
0.6	0.1	0.2	Electric
0.4	0.5	0.2	Steel

Denote the prices (in dollar) of the total annual output of the coal, electric and steel sectors by P_C, P_E and P_S respectively. Find the equilibrium prices that makes each sectors income matches in expenditure.

Or

(b) Find the 3x3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of 90^o about the origin and finally a translation that adds (-0.5, 2) to each point. CO5- App (16)