Reg. No: | | | | | | | | | | | | |

Question Paper Code :R3M26

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Third Semester

Bio technology

R21UMA326-TRANSFORM TECHNIQUES AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to BME and AGRI)

(Regulations R2021)

Duration: Three hours | Maximum: 100 Marks

PART A - (10 x 1 = 10 Marks)

1. If a function f(x) is even, its Fourier expansion contains only ---- terms. CO6-U

(a) Sine (b) Cosine (c) tan (d) None of these

2. The Fourier constant b_n in $(-\pi,\pi)$ for x sin x is _____. CO1-App

(a) x^2 (b)3x (c)0 (d) 1

3. $F_S[x\ f(x)]$ = _____ CO2-App

(a) $-F_c[f(x)]$ (b)$-\frac{d}{ds}\{F_s[f(x)]\}$ (c)$-F_s[f(x)]$ (d) $-\frac{d}{ds}\{F_c[f(x)]\}$

4. If $F[f(x)] = f(s)$ then F[f(ax)] = _____ CO2-App

(a) $\frac{1}{-a}F\left(\frac{s}{a}\right)$ (b) $\frac{1}{a}F\left(\frac{s}{a}\right)$ (c) $\frac{1}{|a|}F\left(\frac{s}{a}\right)$ (d) $\frac{1}{s}F\left(\frac{s}{a}\right)$

5. Z [1] = _____ CO3- App

(a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{z^2}{z+1}$ (d) $\frac{z^2}{z-1}$

6. $Z^{-1}\left[\frac{z}{(z-1)^2}\right]$ = CO3- App

(a) n^2 (b) n^3 (c) n (d) none of these

7. The PDE obtained from $z = (x+a)(y+b)$ is __. CO4-App

(a) $3z = px + qy$ (b) $py - qx = 0$ (c) $z = pq$ (d) $px+qy = 0$

8. The complete solution of $z = px + qy + pq$ is_____. CO4-App

(a) $z = ax+by+ab$ (b) $z = ax-by-ab$ (c) $z= ax+by-ab$ (d) $z = ax-by+ab$

9. Classify the equation $u_{xx}+u_{yy} = 0$ CO6-U

(a) parabolic (b) hyperbolic (c) elliptic (d) cyclic

10. In one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, α^2 is the ____ of the substance. CO6-U

(a) diffusivity (b) specific heat (c) thermal conductivity (d) density

PART – B (5 x 2= 10Marks)

11. Find the root mean square value of the function f(x) = x in (0,l) CO1-App

12. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| < 2 \\ 0 & |x| > 2 \end{cases}$ CO2-App

13. Evaluate $\mathbf{z}\left(3^{\mathbf{n}}\right)$ CO3-App

14. Form the PDE by eliminating arbitrary constants from $z = (x+a)^2 + (y+b)^2$ CO4-App

15. The ends A and B of a rod of length 10cm long have their temperature kept at 20°c & 70°c. Find the steady state temperature distribution on the rod. CO5-App

PART – C (5 x 16= 80Marks)

16. (a) (i) Calculate the Fourier series expansion for $f(x) = (\pi - x)^2$ in $(-\pi, \pi)$ CO1-App (8)

(ii) Express the Fourier Series of $f(x) = x^2$ in $(0, 2\pi)$ CO1-App (8)

Or

(b) The table of values of the function $y = f(x)$ is given below: CO1-App (16)

x	0	T/6	T/3	T/2	2T/3	5T/6	T
y:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x

17. (a) Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{cases}$ and hence deduce that $i) \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ $ii) \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$ CO2-App (16)

Or

(b) (i) Evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+16)(x^2+9)}$ CO2-App (8)

(ii) Find the Fourier Cosine Transform of $\frac{e^{-ax} - e^{-bx}}{x}$ CO2-App (8)

18. (a) (i) Evaluate $Z[\cos n\theta]$ $and Z[\sin n\theta]$ CO3-App (8)

(ii) Evaluate $Z^{-1}\left[\frac{z}{z^2+7z+10}\right]$ using partial fraction method. CO3-App (8)

Or

(b) (i) Using convolution theorem find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z-1)}\right)$ CO3-App (8)

(ii) Solve $y_n + 3y_{n-1} - 4y_{n-2} = 0$ *given* $y(0) = 3, y(1) = -2$ CO3-App (8)

19. (a) (i) Solve $\left(D^2 - 4DD' + 4D'^2\right)z = e^{2x+y} + xy$ CO4-App (8)

(ii) Form a P.D.E by eliminating f from $z = xy + f(x^2 + y^2 + z^2)$ CO4-App (8)

Or

(b) (i) Solve $(mz - ny)p + (nx - lz)q = ly - mx$ CO4-App (8)

(ii) Solve $z = px + qy + p^2q^2$ CO4-App (8)

20. (a) A String is stretched and fastened to two points l apart .Motion is started by displacing the string into the form y= λ x(l-x) from which it is released at t=0.Find the displacement of any point at a distance 'x' at any time 't'. CO5-App (16)

Or

(b) A rod 30cm long has its ends A and B kept at 20^oc and 80^oc respectively until steady state conditions prevails. The Temperature at each end is then suddenly reduced to 0^oc and kept so . Find the resulting temperature for function u(x,t) taking x=0 at A. CO5-App (16)