A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: R2M07

B.E./B.Tech. DEGREE EXAMINATION, NOV 2024

Second Semester

Agricultural Engineering

R21UMA207- CALCULUS COMPLEX ANALYSIS AND TRANSFORM TECHNIQUES

(Regulations R2021)

(Common to Biomedical and Biotechnology engineering branches)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The order and degree of $(y''')^2 + 2(y'')^3 + y = 0$ is _____ CO1- U

(a) 3,2 (b) 2,3 (c) 3,3 (d) 2,2

2. $\frac{1}{D^2}(\cos x) =$ _____ CO1- App

(a)sin x (b) - cos x (c) cos x (d) tan x

3. IF $\vec{F}$ is Solenoidal then $\nabla.\vec{F} =$ CO2- U

(a) 1 (b) 2 (c) 3 (d) 0

4. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is _____ CO2- App

(a) 8 (b)4 (c)-3 (d) 0

5. The critical point of the transformation $w = z + \frac{1}{z}$ are ____ CO3- App

(a) ± 1 (b) ± 2 (c) $\pm i$ (d) $-i$

6. The transformation w=cz is known as _____ CO3- U

(a) Rotation (b) reflection (c) translation (d) rotation and magnification

7. Simple pole is a pole of order _____ CO6- U

(a) 1 (b) 2 (c) 3 (d) 4

8. $\int_C \frac{e^z}{z-2} dz$ where C is the unit circle with centre as origin is _____ CO4- App

(a) 0 (b) 1 (c) 2 (d) π

9. L (e^{at} f(t)) = _____ CO5- App

(a) F(s + a) (b)F(s – a) (c)F(s) (d) – F(s)

10. $L^{-1}(1)$ = _____ CO6- U

(a) t (b) $\frac{1}{t}$ (c) $\frac{1}{s}$ (d) $\delta(t)$

PART – B (5 x 2= 10 Marks)

11. Compute the particular integral of $(D^2 + 7D - 8)y = e^{2x}$ CO1- App

12. Compute $\nabla\varphi$, if $\varphi = x^2 + y^2 + z^2$ at (1, -1, 1). CO2- App

13. Show that the function $f(z) = \bar{z}$ is nowhere differentiable. CO3 -App

14. Compute the Residue of $f(z) = \frac{z+1}{(z-1)(z+2)}$ at z =1. CO4 -App

15. Estimate L[t sint] CO5 -App

PART – C (5 x 16= 80 Marks)

16. (a) (i) Solve $(D^2 - 4D + 3)y = e^{3x} + x^2$ CO1- App (8)

(ii) Solve $[(x+1)^2 D^2 + (x+1)D + 1]y = 4\,Cos\log(x+1)$ CO1-App (8)

Or

(b) (i) Using method of variation of parameters solve CO1-App (8)

$(D^2 + a^2)y = Cosec\ ax$

(ii) A colony of bacteria of growing exponentially. At time t=0 it has 10 bacteria in it and at time t = 4 it has 2000. At what time will it have 100,000 bacteria? CO1-App (8)

17. (a) Verify Stokes theorem for a vector field defined by $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ in the rectangular region in the XOY plane bounded by the lines $x = \pm a, y = 0, and\ y = b.$ CO2-App (16)

Or

(b) Verify Gauss divergence theorem for the vector function $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ and $x = 1, y = 1, z = 1$ CO2-App (16)

18. (a) (i) Determine the analytic function whose real part is $e^x \cos y$ CO3 -App (8)

(ii) Determine the image of the infinite strips (a) $\frac{1}{4} < y < \frac{1}{2}$ CO3 -App (8)

(b) $0 < y < \frac{1}{2}$ under the mapping $w = \frac{1}{z}$

Or

(b) (i) If f(z)=u+iv is an analytic function then Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ CO3- App (8)

(ii) Determine the bilinear transformation that maps the points ∞,i,0 onto 0,i, ∞ respectively CO3 -App (8)

19. (a) (i) Using Cauchy's integral formula Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle |z|= 3 CO4-App (8)

(ii) Evaluate f(z) = $\frac{z-1}{(z+2)(z+3)}$ in Laurent's series valid in the region $2 < |z| < 3$ CO4-App (8)

Or

(b) Using Contour integration , Evaluate $\int_0^{2\pi} \frac{1}{5 + 4\cos\theta} d\theta$ CO4-App (16)

20. (a) (i) Solve the differential equation CO5-App (8)

$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{-t}$ with $y(0) = 1 \& y'(0) = 0$ by using Laplace transform method

(ii) Using Convolution Theorem, Compute $L^{-1}\left[\frac{s}{(s^2+4)^2}\right]$. CO5-App (8)

Or

(b) (i) Compute the Laplace transform of CO5-App (8)

$$f(t) = \begin{cases} k\ , & 0 \le t \le a \\ -k\ , & a \le t \le 2a \end{cases}$$ and f(t +2a) = f(t)

(ii) Using Convolution Theorem, Compute CO5-App (8)

$\mathbf{L^{-1}}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$