A   Reg. No. :

Question Paper Code: U2M02

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Second Semester

Mechanical Engineering

21UMA202 - Calculus, Fourier Series And Numerical Methods

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Gauss Jacobi iteration converges if the coefficient matrix is _____   CO6-U
    dominant.

    (a) Squarely   (b) logically   (c) diagonally   (d) symmetrically

2. Newton's method also called _____ method.   CO6-U

    (a) tangents   (b) slope   (c) secants   (d) false

3. The complementary function of $(4D^2 - 3D - 1)y = 2\sin 2x$ is _____.   CO2-App

    (a)$Ae^{x} + Be^{-\frac{x}{4}}$ (b) $Ae^{-x} + Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d)$Ae^{x} + Be^{4x}$

4. $\dfrac{1}{D^2}(\cos x) = $ _____   CO2-App

    (a) sin x   (b) - cos x   (c) cos x   (d) tan x

5. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is _____.   CO3- App

    (a) 4   (b)– 4   (c)– 3   (d) 0

6. If $\varphi = x^2 + y^2 - z - 10$ then $|\nabla \varphi|$ at (1, 1, 1) is _____ . CO3- App

    (a)$2\vec{i} + 2\vec{j} + \vec{k}$ (b)$2\vec{i} + 2\vec{j} - \vec{k}$ (c) 3   (d) 9

7. cos x is a periodic function with period ----------.   CO4-App

    (a) π   (b) 2 π   (c) π/3   (d) 2 π/3

8. If $f(-x) = -f(x)$, then $f(x)$ is said to be an _____.   CO6-U

   (a) Odd Function   (b) Even Function   (c) Periodic function   (d) Self Reciprocal

9. Convolution theorem on Fourier Transform is $F[f(x)*g(x)]=$ CO6-U

_____

   (a) F(s).G(s)   (b)f(s).g(s)   (c)F(s)*G(s)   (d) f(s)*g(s)

10. $F_s[e^{-ax}] = $ _____   CO5-U

   (a) $\sqrt{\dfrac{2}{\pi}}\dfrac{s}{s^2+a^2}$ (b) $\sqrt{\dfrac{2}{\pi}}\dfrac{a}{s^2+a^2}$ (c) $\sqrt{\dfrac{2}{\pi}}\dfrac{a^2}{s^2+a^2}$ (d) $\sqrt{\dfrac{2}{\pi}}\dfrac{s^2}{s^2+a^2}$

<div align="center">PART – B (5 x 2= 10Marks)</div>

11. State the principle used in Gauss Elimination Method.   CO6-U

12. Find the complete solution of $(D^3 - 6D^2 + 11D - 6)y = 0$ CO2-App

13. Show that $\nabla(r^n)=nr^{n-2}\vec{r}$.   CO3-App

14. Find b_n in the Fourier series of $f(x) = |\cos x|$ in $(0,2\pi)$.   CO4-App

15. Write the Fourier sine transforms pair and Fourier Cosine transform pair.   CO5-App

<div align="center">PART – C (5 x 16= 80Marks)</div>

16. (a) (i) Solve for a positive root of $3x - \cos x - 1=0$ by Newton's   CO1-App   (8)
       Raphson method.
       (ii) Solve $4x + 2y +z = 14$, $x + 5y - z= 10$, $x + y + 8z =20$ by   CO1- App   (8)
       Gauss Elimination method

<div align="center">Or</div>

   (b) (i) Using Power method find numerically largest Eigen value of   CO1- App   (8)

$$\begin{pmatrix} 9 & 1 & 8 \\ 7 & 4 & 1 \\ 1 & 7 & 9 \end{pmatrix}$$

       (ii) Solve $27x + 6y - z = 85$, $6x + 15y +2z = 72$, $x + y + 54z =110$ CO1- App   (8)
       by Gauss Seidel method

17. (a) (i) Using method of variation of parameters solve   CO2-App   (8)
       $(D^2 + a^2)y = \csc ax$

       (ii) Solve $(D^2 + 2D + 2)y = \cos 2x$ CO2-App   (8)

<div align="center">Or</div>

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(b) (i) Solve $(D^2 - 3D + 2)y = 2e^x + 2\cos 2x$ CO2 -App (8)

(ii) If the population of a country double in 50 years, in how CO2 -App (8)
many years will it triple under the assumption that the rate of
increase of proportional to the number of inhabitants?

18. (a) Verify   Gauss   Divergence   theorem   for CO3-App (16)
$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ over   the   rectangular
parallelepiped $x = 0, x = a, y = 0, y = b, z = 0, z = c$

Or

(b) (i) Prove that $\bar{F} = (x^2 + xy^2)\bar{\iota} + (y^2 + x^2 y)\bar{\jmath}$ is irrotational CO3-App (8)
vector and find the Scalar potential such that $\bar{F} = \nabla\emptyset$.

(ii) Evaluate Green's theorem for $\int (x^2 - y^2)dx + 2xydy$, where CO3-App (8)
C is bounded by $x = 0, x = a,\ \ y = 0\ \ and\ \ \ y = b$

19. (a) Find the Fourier series of $f(x) = |\sin x|\ in - \pi < x < \pi$ of periodicity CO4-App (16)
2π.

Or

(b) (i) The table of values of the function $y = f(x)$ is given below: CO4-App (8)

x:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

Find a Fourier series up to the third harmonic to represent $f(x)$
in terms of x.

(ii) Find the half range sine series for $f(x) = x^2$ in $(0, l)$ CO4-App (8)

20. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & if \quad |x| \le 1 \\ 0 & if \quad |x| > 1 \end{cases}$ and hence CO5-App (16)

deduce that i) $\int_0^\infty \left(\dfrac{\sin t}{t}\right)^2 dt = \dfrac{\pi}{2}$.

Or

(b) (i) Find Fourier Cosine and Sine Transform of e^{-ax} . CO5- App (8)

(ii) Evaluate $\int_0^\infty \dfrac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ CO5- App (8)

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