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Reg. No. :

Question Paper Code: 53021

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2022

Third Semester

Civil Engineering

15UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, EIE, MECH, Chemical, Biomedical and

Agriculture Engineering Branches)

(Regulation 2015)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The constant term in the Fourier series is   CO1- R

    (a) a_0 (b) b_1 (c) a_5 (d) b_4

2. The root mean square value of $f(x) = x$ in $(0, 1)$ interval   CO1- R

    (a) 2/3   (b) $1/(3)^{1/2}$ (c) $2/(3)^{1/2}$ (d) 4/5

3. Fourier integral of $f(x) = 1$. $0 < x < \infty$ CO2- R

    (a) 0   (b) 1   (c) Not defined   (d) Very large number

4. Give a function which is self reciprocal under sine transform   CO2- R

    (a) x   (b) x^2 (c) $1/(x)^{(1/2)}$ (d) $1/(x)^{(3/2)}$

5. Find $z\left[a^{n-1}\right]$ CO3- R

    (a) $\dfrac{az}{z-1}$ (b) $\dfrac{1}{z-1}$ (c) $\dfrac{z^2}{z-a}$ (d) $1/a\left(\dfrac{z}{z-a}\right)$

6. Find   CO3- R

$$Z^{-1}\left[\frac{z}{z+1}\right]$$

   (a) $(-1)^n$ (b) $(-a)^n$ (c) $(-t)^n$ (d) $(1)^n$

7. The p.d.e of z = ax+by is   CO4- R

   (a) x+y   (b) qx+py   (c) px+qy   (d) x-y

8. Find the P.I of $[D^2 + 4DD']z = e^x$ CO4- R

   (a) 1   (b) e^x (c)0   (d) e^{x-1}

9. What is the constant a^2 in the wave equation   CO5- R

   (a) $a^2 = \dfrac{T}{m}$ (b) $a^2 = \dfrac{1}{m}$ (c) $a^2 = \dfrac{T}{2}$ (d) $a^2 = \dfrac{T^2}{m}$

10. Governing equation of two dimensional steady state heat equation is   CO5- R

   (a) $\dfrac{\partial u}{\partial x} + \dfrac{\partial^2 u}{\partial y^2} = 0$ (b) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 1$ (c) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial u}{\partial y} = 0$ (d) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

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PART – B (5 x 2= 10 Marks)

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11. Explain Dirichlet's condition   CO1- R

12. State the Convolution theorem for Fourier Transforms   CO2- R

13. Define Difference equations   CO3- R

14. From the p.d.e by eliminating arbitrary constants a and b from   CO4- R

   $z = (x+a)^2 + (y-b)^2$

15. Write all variable separable solutions of the one dimension heat equation.   CO5- R

16. (a) Find the Fourier series of x^2 in $(-\pi, \pi)$. Hence prove the following   CO1- App   (16)

$(a) \dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots = \dfrac{\pi^2}{6}$

$(b) \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \cdots = \dfrac{\pi^2}{12}$

$(c) \dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \cdots = \dfrac{\pi^4}{90}$

Or

(b) Determine the first two harmonic of the Fourier series for the following values.   CO1- App   (16)

X:	0	$\dfrac{\pi}{3}$	$\dfrac{2\pi}{3}$	π	$\dfrac{4\pi}{3}$	$\dfrac{5\pi}{3}$
Y:	1.98	1.30	1.05	1.30	-0.88	-0.25

17. (a) Find the Fourier transform of   CO2- App   (8)

$$f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}.$$

Hence deduce the following:

$(i) \displaystyle\int_0^\infty \left(\dfrac{\sin t}{t}\right)^2 dt = \dfrac{\pi}{2}$ CO2- App   (4)

$(ii) \displaystyle\int_0^\infty \left(\dfrac{\sin t}{t}\right)^4 dt = \dfrac{\pi}{3}$ CO2- App   (4)

Or

(b) Show that   CO2- App   (16)

$e^{-x^2/2}$ is self reciprocal under Cosine Transform.

18. (a) Find   CO3- Ana   (4)

    (i) $Z[a^n \cos n\theta]$

    (ii) $Z[\sin n\theta]$ CO3- Ana   (4)

    (iii) Using convolution theorem, evaluate the inverse   CO3- Ana   (8)
    Z – transform of $\dfrac{z^2}{(z-a)(z-b)}$

Or

(b) Solve   CO3- Ana   (16)

   $y_{n+2} + 6\,y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$, using Z – transform.

19. (a) (i) Find the singular integral of $z = px + qy + p^2 + pq + q^2$ CO4-App   (8)

    (ii) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ CO4-App   (8)

Or

(b) Solve $(D^2 + 2DD' + D'^2)z = x^2 y + e^{x-y}$.   CO4- App   (16)

20. (a) A tightly stretched flexible string has its ends fixed at x = 0 and   CO5- U   (16)
    x = ℓ. At time t = 0, the string is given a shape defined by
    f (x) = $kx(\ell - x)$, where 'k' is constant and then released from
    rest. Find the displacement of any point 'x' of the string at any
    time t > 0.

Or

(b) An insulated rod of length l has its ends A and B maintained   CO5- U   (16)
    at 0^0 C and 100^0 c respectively until steady state conditions prevail.
    If B is suddenly reduced to 75^0 C and at A raised to 25^0 C, find
    the temperature at a distance x from A at time t.

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