A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U3028

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Third Semester

Computer Science and Design

21UMA328- DISCRETE MATHEMATICS

(Common to Aritificial Intelligence & Data Science Engineering)

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. What is the value of x after this statement, assuming the initial value of x is 5? CO6-U
If x equals to one then x=x+2 else x=0'.

(a) 1 (b) 3 (c) 0 (d) 2

2. The statement," Every comedian is funny" where C(x) is "x is a comedian" CO1- App
and F (x) is "x is funny" and the domain consists of all people.

(a)$\exists x(C(x) \wedge F(x))$ (b) $\forall x(C(x) \wedge F(x))$ (c)$\exists x(C(x) \rightarrow F(x))$ (d) $\forall x(C(x) \rightarrow F(x))$

3. What is the base case for the inequality $7n > n^3$, where n = 3? CO2- App

(a) $652 > 189$ (b)$42 < 132$ (c)$343 > 27$ (d) $42 \leq 431$

4. The numbers between 1 and 520, including both, are divisible by 2 or 6 is CO2- App
_____.

(a) 349 (b)54 (c) 213 (d) 303

5. A group (M, *) is said to be abelian if _____ CO6- U

(a) (x + y) = (y + x) (b) (x * y) = (y * x) (c) (x + y) = x (d) (y * x) = (x + y)

6. Intersection of subgroups is a _____. CO6- U

(a) group (b) subgroup (c) semi group (d) cyclic group

7. In a _____ the degree of each and every vertex is equal. CO6- U

(a) regular graph (b) point graph (c) star graph (d) Euler graph

8. Every complete bipartite graph must not be _____. CO6- U

(a) planar graph (b) line graph (c) complete graph (d) subgraph

9. Which of the following Law of Boolean proofs the X. X=X? CO6- U

(a) Identity Law (b) Double Complement Law

(c) Complement Law (d) Idempotent Law

10. A _____ is a Boolean variable. CO6- U

(a) Literal (b) String (c) Keyword (d) Identifier

PART – B (5 x 2= 10Marks)

11. Find the value of the premises (p ∧ q) ∨ r and r → s CO1- App

12. What is the generating function for generating series 1, 2, 3, 4, 5, …? CO2- App

13. Define cyclic group with an example. CO6- U

14. Draw K_5 complete graph. CO6- U

15. State distributive lattice. CO6- U

PART – C (5 x 16= 80Marks)

16. (a) (i) Find PDNF and PCNF of$(P \wedge Q) \vee (\neg P \wedge R)$. CO1 -App (8)

(ii) Prove that $((p \vee q) \wedge \neg (\neg p \wedge (\neg q \vee \neg r))) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is a tautology. CO1 -App (8)

Or

(b) (i) Prove that A$\longrightarrow \neg D$ is a conclusion from the premises $A \longrightarrow B \vee C$, $B \longrightarrow \neg A and D \longrightarrow \neg C$by using conditional proof. CO1 -App (8)

(ii) Show that that $\exists x P(x) \longrightarrow \forall x Q(x) \Rightarrow \forall x\ (P(x) \longrightarrow Q(x))$. CO1 -App (8)

17. (a) (i) Solve:$a_{n+2} + 3a_{n+1} + 2a_n = 3^n, a_0 = 0, a_1 = 1$ CO2 -App (8)

(ii) Using generating functions Solve, CO2 -App (8)

$a_n - 3a_{n-1} + 2a_{n-2} = 0, n \geq 2 a_0 = 2, a_1 = 3.$

Or

(b) (i) Prove that by using mathematical Induction $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\ldots+\frac{1}{n(n+1)}=\frac{n}{(n+1)}$ CO2 -App (8)

(ii) Out of 100 students in a college, 38 play tennis, 57 play cricket and 31 play hockey, 9 play cricket and hockey, 10 play hockey and tennis, 12 play tennis and cricket. CO2 -App (8)

How many plays (1) All three games.

(2) Atleast two game.

(3) Hockeyor cricket but not tennis

(Assume that each student plays atleast one game.)

18. (a) (i) Show that$(Q^+,*)$ is an abelian group where $*$ is defined as $a*b = ab/2$, $\forall a, b \in Q^+$. CO3- App (8)

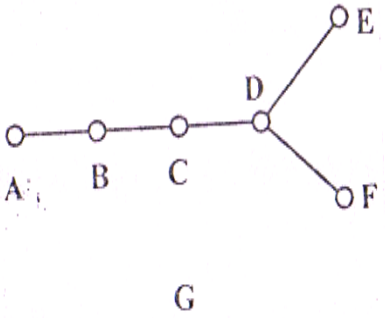
(ii) S.T by using an example "the union of two subgroup of a group G need not be a subgroup". CO3- App (8)

Or

(b) (i) State and prove Lagrange's theorem. CO3- App (8)

(ii) Prove that $(\boldsymbol{a}*\boldsymbol{b})^2 = \boldsymbol{a}^2*\boldsymbol{b}^2$ (ffy io abelian) CO3- App (8)

19. (a) (i) Verify that following are isomorphic graph are not CO4-App (8)



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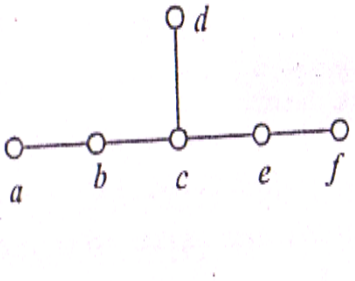

G'

(ii) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. CO4-App (8)

Or

(b) (i) Define Isomorphism between the two graphs. Are the simple graphs with the following adjacency matrices isomorphic? CO4-App (8)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) Prove that a given connected graph is Eulerian if and only if all the vertices of G are of even degree. CO4-App (8)

20. (a) (i) State and prove the distributive inequalities of a Lattice. CO5- App (8)

(ii) State and prove De Morgan's law for Boolean algebra. CO5- App (8)

Or

(b) (i) Show that in a complemented distributive lattice, the De Morgan's laws hold good. CO5- App (8)

(ii) Show that in any Boolean algebra $(a + b)(a' + c) = ac + a'b + bc = ac + a'b$ CO5- App (8)

$$\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xy\,dx\,dy$$