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Question Paper Code: 93027

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Third Semester

Computer Science and Business System

19UMA327- Discrete Mathematics and Calculus

(Regulation 2019)

Duration: Three hours

Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. $P \vee \neg P$ is a CO6-U
 (a) Tautology (b) Contradiction (c) Contingency (d) PDFN

2. Contra positive of $P \rightarrow Q$ CO6-U
 (a) $\neg Q \rightarrow \neg P$ (b) $\neg P \rightarrow \neg Q$ (c) $Q \rightarrow P$ (d) None of these

3. How many ways can letters of the word "GOOGLE" be arranged CO2- App
 (a) 40 (b) 150 (c) 160 (d) 180

4. If 'm' Pigeon occupies 'n' ($m > n$) holes then atleast one hole has more than ---- CO6- U
 Pigeons
 (a) $\left[\frac{n-1}{m} \right]$ (b) $\left[\frac{m-1}{n} \right]$ (c) $\left[\frac{m-1}{n} \right] + 1$ (d) $\left[\frac{n-1}{m} \right] + 1$

5. The order of group $G = \{1, -1, i, -i\}$ under usual multiplication CO3- App
 (a) 4 (b) 0 (c) 1 (d) 2

6. $(\mathbb{N}, +)$ is a CO6- U
 (a) Sub group (b) semi group (c) group (d) Moniod

7. Find $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ CO4- App
 (a) 1 (b) 2 (c) 3 (d) 4

8. Find the $\frac{dy}{dx}$ for $\cos \sqrt{x}$ CO4- App

- (a) $-\sin \sqrt{x}$ (b) $\frac{-\sin \sqrt{x}}{2\sqrt{x}}$ (c) $-\cos \sqrt{x}$ (d) $\frac{\cos \sqrt{x}}{2\sqrt{x}}$

9. $\iint_R dx dy$ gives _____ CO6- U

- (a) Volume of R (b) Area of the region R (c) Length of R (d) None of these

10. The value of $\int_1^b \int_1^a \frac{dx dy}{xy}$ CO5- App

- (a) $\log a + \log b$ (b) $\log a$ (c) $\log b$ (d) $\log a \log b$

PART – B (5 x 2= 10Marks)

11. Construct the truth table for $(p \vee \neg q) \rightarrow q$ CO1- App

12. Find the number of positive integers not exceeding 100 that are divisible by 5 or by 11 CO2- App

13. In an abelian group prove that $(ab)^2 = a^2 b^2$ CO3-App

14. Solve $\int \frac{dx}{x^2 - 6x + 13}$ CO4- App

15. Change the order of integration $\int_0^a \int_y^a f(x, y) dx dy$ CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Prove the following using Indirect method . CO1- App (8)

$P \rightarrow Q, Q \rightarrow R, \neg P \vee \neg R, P \vee R \Rightarrow R$

(ii) Using truth table find PCNF and PDNF for CO1- App (8)

$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Or

(b) (i) Show that the premises “one student in this class knows how to write programs in JAVA” and “Every one who knows how to write programs in JAVA can get a high- paying job” imply the conclusion “some one in this class can get high paying job” CO1 - App (8)

(ii) Using the rules of inference derive CO1 -App (8)

$P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S)$

17. (a) (i) Find the number of positive integers between 1 and 600 divisible by 2, 3, 5 or 7 CO2 -App (8)
- (ii) Using Mathematical Induction show that, $n^3 + 2n$ is divisible by 3 CO2 -App (8)
- Or
- (b) (i) There are seven men and six women in a room. Find the number of ways four persons can be drawn from the room if CO2 -App (8)
- (a) they can be male or female,
 (b) two must be men and two women,
 (c) they all are of the same Gender.
- (ii) Using generating functions Solve $a_n = 2a_{n-1} + 2^n, a_0 = 2$ CO2 -App (8)
18. (a) (i) Let G be a finite group of order 'n' and H be any subgroup of G CO3- App (8)
 Then the order of H divides the order of G. (i.e) $O(H) / O(G)$
- (ii) The intersection of two subgroup of a group is also a subgroup of the group CO3- App (8)
- Or
- (b) (i) Prove that in a group G is abelian iff $(a * b)^2 = a^2 * b^2$ CO3- App (8)
- (ii) Prove that the union of two subgroup of G needs not a subgroup CO3- App (8)
19. (a) (i) If $y = e^{ax} \cos bx$ Prove that $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$ CO4-App (8)
- (ii) Determine the reduction formula for $\int \sin^n x dx$ CO4-App (8)
- Or
- (b) (i) Compute $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\cos x)^{\frac{3}{2}} + (\sin x)^{\frac{3}{2}}} dx$ CO4 -App (8)
- (ii) If $y = (2 \cos t - \cos 2t), x = (2 \sin t - \sin 2t)$ Find the value of $\frac{d^2y}{dx^2}$ CO4 -App (8)
- at $t = \left\{ \frac{\pi}{2} \right\}$

20. (a) (i) Compute the volume of the Sphere $x^2 + y^2 + z^2 = a^2$ without transformation CO5- App (8)

(ii) Change the order of integration and hence evaluate CO5- App (8)

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{\sqrt{ax}} xy \, dy \, dx$$

Or

(b) (i) Using the Triple integration, compute the volume of the CO5- App (8)

tetrahedron $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, x=0, y=0 \text{ \& } z=0$

(ii) Evaluate CO5- App (8)

$$\iiint \frac{dx dy dz}{\sqrt{a^2 - x^2 - y^2 - z^2}} \quad \text{over the first octant of the sphere}$$

$$x^2 + y^2 + z^2 = a^2$$