Question Paper Code: 50002

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2022

First Semester

Civil Engineering

15UMA102 – ENGINEERING MATHEMATICS - I

(Common to ALL Branches)

(Regulation 2015)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

    (a) 0   (b) 4   (c) 1   (d) 6

2. If $x = a\cos\theta$, $y = b\sin\theta$, then $\frac{dy}{dx}$ is

    (a) $\frac{-b}{a}\cot\theta$ (b) $\frac{b}{a}\cot\theta$ (c) $b\cos\theta$ (d) $b\sin\theta$

3. If U is a homogeneous function of degree 'n' then by Euler's theorem

    (a) $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = U$ (b) $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = nU$

    (c) $x\frac{\partial U}{\partial x} - y\frac{\partial U}{\partial y} = U$ (d) $x\frac{\partial U}{\partial x} - y\frac{\partial U}{\partial y} = nU$

4. The order of $f(x, y) = \frac{x^6 + y^6}{x^4 - y^4}$ is

    (a) 3   (b) 2   (c) 1   (d) 0

5. $\int x e^x dx$ is

    (a) $e^x(x + 1) + 1$ (b) $e^x(x - 1)$
    (c) $e^x(1 - x)$ (d) xe^x

6. If $f(x)$ is odd then $\int_{-1}^{1} f(x)dx$.

    (a) 2   (b) 0   (c) 1   (d) 1/2

7. Change the order of integration of $\int_0^a \int_y^a f(x,y)dxdy$ is

    (a) $\int_0^x \int_0^a f(x,y)dydx$ (b) $\int_0^a \int_0^x f(x,y)dydx$

    (c) $\int_0^a \int_a^x f(x,y)dydx$ (d) $\int_0^a \int_0^y f(x,y)dydx$

8. $\int_0^1 \int_0^2 \int_0^3 dzdydx$ is

    (a) 0   (b) 1   (c) 2   (d) 6

9. The eigen values of $\begin{bmatrix} 5 & 6 & 17 \\ 0 & -9 & 23 \\ 0 & 0 & 37 \end{bmatrix}$ are

    (a) 5, -9, 23   (b) 6, -9, 37   (c) 17, 6, 5   (d) 5, -9, 37

10. Matrix $\begin{bmatrix} x & 2 \\ 1 & x-1 \end{bmatrix}$ is singular for $x =$

    (a) 1, 2   (b) -1,-2   (c) -1, 2   (d) 1,-2

<div align="center">PART - B (5 x 2 = 10 Marks)</div>

11. If $x^3 + y^3 = 3axy$ then find $\frac{dy}{dx}$.

12. FIf $x = r\cos\theta, y = r\sin\theta, then\ find\ J(u,v)$.

13. Evaluate $\int_0^{\pi/2} \sin^6 x dx$.

14. Evaluate $\int_0^1 \int_1^2 x(x+y)dydx$.

15. Define Cayley Hamilton theorem and its applications.

<div align="center">PART - C (5 x 16 = 80 Marks)</div>

16. (a) If y= $e^{a\sin^{-1}x}, prove\ that$

    $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ (16)

<div align="center">Or</div>

    (b) Find the nth derivative of $\frac{x^3}{(x-a)(x-b)(x-c)}$ (16)

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17. (a) Investigate the maxima of the function $f(x, y) = x^3 y^2 (1 - x - y)$. (16)

Or

(b) A rectangular box open at the top is said to have a volume of 32cc. Find the dimensions of the box that requires the least material for its construction. (16)

18. (a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (16)

Or

(b) (i) Prove that $\Gamma(n + 1) = n\Gamma(n)$. (8)

(ii) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m, m)$. (8)

19. (a) Change the order of integration and then evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$. (16)

Or

(b) Evaluate $\int \int \int (x + y + z) dx \, dy \, dz$ over the region V, where the region V is bounded by x + y + z = a. x = 0, y = 0, z = 0. (16)

20. (a) Verify Cayley – Hamilton theorem for $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ Hence find A^{-1} and A^4. (16)

Or

(b) Reduce the Quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1 x_2 + 2x_2 x_3$ to the canonical form through an orthogonal transformation and hence show that it is positive semi definite. Give also a non-zero set of values which will make the quadratic form zero. (16)

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