A

Question Paper Code: U3025

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Third Semester

Civil Engineering

21UMA325- PROBABILITY, STATISTICS AND TRANSFORM TECHNIQUES

(Regulations 2021)

Duration: Three hours   Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. Probability of an impossible event is   CO6-U

    (a) 1   (b) 10   (c) 0   (d) 100

2. The mean of the random variable is denoted by   CO6- U

    (a) $E(X)$ (b) $E(X^2)$ (c) 0   (d) 1

3. The degrees of freedom in t-tests is   CO6- U

    (a) n-1   (b) n-2   (c) n-3   (d) n-4

4. Choose the t-test for mean   CO6- U

    (a) $t = \dfrac{\overline{x_1} - \mu}{s / \sqrt{n - 1}}$ (b) $t = \dfrac{\overline{x_1} + \mu}{s / \sqrt{n - 1}}$ (c) $t = 0$ (d) None of the above

5. If $f(x + t) = f(x)$, then $f(x)$ is said to be an _____   CO6- U

    (a) Odd Function   (b) Even Function   (c) Periodic function   (d) Self Reciprocal

6. The Fourier constant b_n in $(-\pi,\pi)$ for $x \sin x$ is _____   CO6- U

    (a) x^2 (b) 3x   (c) **0**   (d) 1

7. $F[xf(x)] =$ _____   CO6- U

    (a) $-F_c[f(x)]$ (b) $-\dfrac{d}{ds}\{F_s[f(x)]\}$ (c) $-F_s[f(x)]$ (d) $-\dfrac{d}{ds}\{F_c[f(x)]\}$

8. $F_s[e^{-ax}] = $ _____   CO6- U

(a) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (b) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (c) $\sqrt{\frac{2}{\pi}}\frac{a^2}{s^2+a^2}$ (d) $\sqrt{\frac{2}{\pi}}\frac{s^2}{s^2+a^2}$

9. The Z transform of a unit step function is _____.   CO5- U

(a) $\log(\frac{z}{z+1})$ (b) $\frac{z}{z+1}$ (c) $\log(\frac{z}{z-1})$ (d) $\frac{z}{z-1}$

10. Evaluate $Z(\frac{1}{n!})$ CO5- U

(a) $e^{-1/z}$ (b) $e^{1/z}$ (c) e^{2z} (d) $e^{1/z} - 2$

PART – B (5 x 2= 10Marks)

11. If $f(x) = \begin{cases} Kxe^{-x} & , x > 0 \\ o & , elsewhere \end{cases}$ is the PDF of a RV X, Find K   CO1- App

12. Write the important properties of F-distribution   CO2- U

13. Find b_n in the Fourier series of $f(x) = |\cos x|$ in $(0,2\pi)$.   CO3- App

14. State Fourier integral theorem.   CO6- U

15. Prove that $Z(sin\frac{n\pi}{2})=\frac{z}{z^2+1}$ CO5 App

PART – C (5 x 16= 80Marks)

16. (a) A Random Variable X has the following probability distribution   CO1-App   (16)

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find   (i) 'k'

(ii) $P(X < 6), P(X \geq 6)$ & $P(1.5 < X < 4.5 / X > 2)$

(iii) If $P(X \leq k) > \frac{1}{2}$ find the minimum value of 'k'

(iv) Distribution function of x

Or

(b) (i) The number of monthly breakdowns of a computer is a R.V. CO1 -App   (8)
having a Poisson distribution with mean equal to 1.8. Find the
Probability that his computer will function for a   month   (a)
Without a breakdown (b) With only one breakdown (c) With at
least one breakdown

(ii) Using an Geometric distribution State and Prove the memory   CO1 -App   (8)
less property.

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17. (a) (i) A sample analysis of examination results of 500 students was made. It was found that 220 students have failed, 170 have secured a third class, 90 have secures a second class and the rest, a first class. So these figures support the general belief that the above categories are in the ratio 4:3:2:1 respectively?   CO2 -Ana   (8)

(ii) Two group of students A and B were tested , the marks obtained were as follows   CO2 -Ana   (8)

| A | 18 | 20 | 36 | 50 | 49 | 36 | 34 | 49 | 41 |
| B | 29 | 28 | 26 | 35 | 30 | 44 | 46 | | |

Examine the significance of difference between the average marks secured by the students of the above two groups

Or

(b) (i) Two horses A and B were tested according to time (in seconds) to run on a particular track with the following results: CO2 -Ana   (8)

| Horse A | 28 | 30 | 32 | 33 | 33 | 29 | 34 |
| Horse B | 29 | 30 | 30 | 24 | 27 | 29 | |

Test whether horse A is running faster than B at 5% level.

(ii) A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight.   CO2 -Ana   (8)

| Diet A | 5 | 6 | 8 | 1 | 12 | 4 | 3 | 9 | 6 | 10 |
| Diet B | 2 | 3 | 6 | 8 | 10 | 1 | 2 | 8 | | |

Find the variances are significantly different

18. (a) Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$.   CO3- App   (16)

Or

(b) The table of values of the function $y = f(x)$ is given below: CO3- App   (16)

| x: | 0 | $\pi/3$ | $2\pi/3$ | π | $4\pi/3$ | $5\pi/3$ | 2π |
| y: | 1.8 | 0.3 | 0.5 | 2.16 | 1.3 | 1.76 | 1.8 |

Find a Fourier series up to the third harmonic to represent $f(x)$ in terms of x

19. (a) Compute the Fourier Transform of $f(x) = \begin{cases} 1 - x^2 & |x| < 1 \\ 0 & \text{otherwise} \end{cases}$ CO4-App   (16)

and hence evaluate the value of (i) $\int\limits_0^\infty \dfrac{\sin t - t\cos t}{t^3} \, dt$

$\int\limits_0^\infty \left(\dfrac{\sin t - t\cos t}{t^3} \right)^2 dt$

Or

(b) Compute (i) $\int\limits_0^\infty \dfrac{dx}{(x^2+1)(x^2+4)}$ (ii) $\int\limits_0^\infty \dfrac{x^2 \, dx}{(x^2+9)^2}$ using Fourier   CO4-App   (16)

transform

20. (a) (i) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ CO5- App   (8)
given that $y_0 = 0$, $y_1 = 0$

(ii) Using Convolution theorem find $Z^{-1}\left[\dfrac{8z^2}{(4z-3)(2z+1)} \right]$ CO5- App   (8)

(b) (i) Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 2^n$ CO5- App   (8)
given that $y_0 = 0$, $y_1 = 0$

(ii) Evaluate $Z[\cos n\theta] \text{ and } Z[\sin n\theta]$ CO5- App   (8)

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