A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U3026

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Third Semester

Agriculture Engineering

21UMA326- TRANSFORM TECHNIQUES AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Biomedical and Biotechnology Engineering)

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. The term ($a_1 \cos x + b_1 \sin x$) in fourier series is called _____ CO6-U

   (a) First harmonic (b) Second harmonic (c) Third harmonic (d) Fourier Coefficients

2. Cos x is a periodic function with period _____ CO6- U

   (a) π (b) 2π (c) $\pi/3$ (d) $2\pi/3$

3. If $F[f(x)] = f(s)$ then the function is said to be _____ CO6- U

   (a) Odd (b) Even (c) Self-Reciprocal (d) Periodic

4. If $F[f(x)] = f(s)$ thenF[f(ax)] = _____ CO2- App

   (a) $\frac{1}{-a} F\left(\frac{s}{a}\right)$ (b) $\frac{1}{a} F\left(\frac{s}{a}\right)$ (c) $\frac{1}{|a|} F\left(\frac{s}{a}\right)$ (d) $\frac{1}{s} F\left(\frac{s}{a}\right)$

5. If $Z\{f(t)\} = F(Z)$, then $Z\{e^{-at}f(t)\}$ = _____ CO6- U

   (a) $F[e^{aT}]$ (b) $F[Ze^{aT}]$ (c) $F[Ze^{-aT}]$ (d) $F[e^{-aT}]$

6. $Z[n f(n)]$ = _____ CO6- U

   (a) $-z\frac{d}{dz}F(Z)$ (b) $z\frac{d}{dz}F(Z)$ (c) $\frac{d}{dz}F(Z)$ (d) $z^n\frac{d}{dz}F(Z)$

7. The PDE obtained from z = (x+a)(y+b) is __. CO4- App

   (a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

8. The particular integral of $(D^2 - 4DD' + 3D'^2)\, z = e^{x+y}$ is _____ CO4- App

(a) $\frac{xe^{x+y}}{2}$ (b) $-\frac{xe^{x+y}}{2}$ (iii) $\frac{x^2e^{x+y}}{2}$ (iv) $\frac{-x^2e^{x+y}}{2}$

9. Classify the equation y2uxx+uyy = 0 is _____ CO6- U

(a) parabolic (b) hyperbolic (c) elliptic (d) cyclic

10. An insulated rod of length 60 cm has its ends at A and B kept at 20oC and 80oC respectively, then its steady state solution is CO5- App

(a) x-20 (b) 4x+20 (c) x+20 (d) x+60

PART – B (5 x 2= 10Marks)

11. Find $a_0\ and a_n$ in the Fourier series of $f(x) = x + x^3$ in (-π,π) CO1- App

12. Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ CO2- App

13. Find $Z\left[\sin\left(\frac{n\pi}{2}\right)\right]$ CO3- App

14. Solve: $\left(D^2 - 4DD' + 4D'^2\right)Z = \sin(x+y)$. CO4- App

15. Write the three Possible solutions of the one dimensional wave equations CO5- U

PART – C (5 x 16= 80Marks)

16. (a) (i) Find the Fourier series of $f(x) = \begin{cases} -1+x, & -\pi < x < 0 \\ 1+x, & 0 < x < \pi \end{cases}$ of periodicity 2π. CO1 -App (8)

(ii) Find the Half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. CO1 -App (8)

Deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} \ldots\ldots = \frac{\pi^4}{90}$

Or

(b) The table of values of the function $y = f(x)$ is given below: CO1 -App (16)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Find a Fourier series upto the third harmonic for $f(x)$in terms of x.

17. (a) Find the Fourier transform of $\mathbf{f(x)} = \begin{cases} \mathbf{a - |x|,} & \mathbf{if} \quad |x| \le a \\ \mathbf{0} & \mathbf{if} \quad |x| > a \end{cases}$ and CO2 -App (16)

hence deduce that $(i)\int_0^{\infty}\left(\frac{\sin t}{t}\right)^2 dt$ (ii) $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^4 dt$

Or

(b) Evaluate (i) $\int_0^{\infty}\frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ (ii) Evaluate $\int_0^{\infty}\frac{dx}{(x^2+49)^2}$ CO2 -App (16)

18. (a) (i) Solve the difference equation $y_{n+2}+6y_{n+1}+9y_n = 2^n$ given that $y_0 = 0,\ y_1 = 0$ CO3- App (8)

(ii) Using Convolution theorem find $Z^{-1}\left[\frac{8z^2}{(4z-3)(2z+1)}\right]$ CO3- App (8)

Or

(b) (i) Solve the difference equation $y_{n+2}+4y_{n+1}+3y_n = 2^n$ given that $y_0 = 0,\ y_1 = 0$ CO3- App (8)

(ii) Evaluate $Z[r^n\cos n\theta]$ *and* $Z[r^n\sin n\theta]$. CO3- App (8)

19. (a) (i) Solve $x(z^2-y^2)p + y(x^2-z^2)q = z(y^2-x^2)$ CO4-App (8)

(ii) Solve $(D^2 - DD' - 2D'^2)z = 2x+3y+e^{3x+4y}$ CO4-App (8)

Or

(b) (i) Solve $x(y-z)p + y(z-x)q = z(x-y)$ CO4-App (8)

(ii) Solve $(D^2 - 7DD' + 6D'^2)z = e^{2x+y}$ CO4-App (8)

20. (a) A String is stretched and fastened to two points l apart .Motion is started by displacing the string into the form $y=K(lx-x^2)$ from which it is released at t=0.Find the displacement of any point at a distance 'x' at any time 't'. CO5- App (16)

(b) A tightly String with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If its set vibrating giving each point at velocity $\lambda(lx-x^2)$.Find the displacement . CO5- App (16)