A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U3027

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Third Semester

Computer Science and Business system

21UMA327- DISCRETE MATHEMATICS AND CACULUS

(Regulations 2021)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. The truth value "If 71 is prime then 3 is even" , The truth value "1 > 3 and 3 is a positive integer " CO1-U

   (a) T,F (b) F,T (c) T,T (d) F,F

2. $P \rightarrow \neg Q$ is equivalent to CO6- U

   (a) $\neg P \wedge Q$ (b) $P \wedge \neg Q$ (c) $\neg(P \wedge Q)$ (d) $P \vee \neg Q$

3. If a bit string contains {0, 1} only, having length 5 has no more than 2 ones in it. Then calculate how many such bit strings are possible? CO2- App

   (a)14 (b)12 (c)16 (d)12

4. Calculate how many integers between 1 to250 are divisible by 2 or 3 CO2- App

   (a) 41 (b)167 (c)83 (d) 174

5. A subgroup of the group $\{1, \omega, \omega^2\}$ where $\omega^3 = 1$ under the multiplication is CO6- U

   (a) $\{1, \omega\}$ (b) $\{\omega, \omega^2\}$ (c) $\{1, \omega^2\}$ (d) None of the above

6. The union of two subgroup of G is a CO6- U

   (a) Subgroup (b) semi group (c) group (d) Monoid

7. $\int_0^\infty e^{-x} x^4 dx$ CO4- App

   (a) 4 (b) 4! (c) 5 (d) 5!

8. $\int_0^{\infty} 6e^{-x}x^5\,dx$ CO4- App

(a) 6 (b) 6! (c) 7! (d) 5!

9. The region of integration of the integral $\int_0^1\int_0^x f(x,y)\,dxdy$ is CO6- U

(a) square (b) rectangle (c) triangle (d) circle

10. The value of integral $\int_1^2\int_1^4 \frac{dxdy}{xy}$ CO5- App

(a) $\log 8$ (b) $(\log 2)^2$ (c) $\log 6$ (d) None of the above

PART – B (5 x 2= 10Marks)

11. Compute PDNF for $(P \vee Q)$ CO1- App

12. In how many ways can letters of the word "THUNAIEZHUTHU" be arranged CO2- App

13. For a Group $G = \{1, -1, -i, i\}$ under multiplication ,Find order of all elements CO3- App

14. Compute y_{25} if $y = \frac{1}{x}$ CO4- App

15. Solve $\int_0^1\int_0^2 x^2y^2\,dydx$ CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Calculate PCNF and PDNF for $(P \wedge \neg Q) \vee (P \wedge R) \vee (Q \wedge R)$ CO1 -App (8)

(ii) Using the rules of inference derive & using CP Rule. CO1 -App (8)

$P \to (Q \to V), \neg U \vee P, Q \Rightarrow U \to (V \wedge P)$

Or

(b) (i) Prove the following by Indirect Method. CO1 -App (8)

$P \to (Q \wedge R),\ (Q \vee S) \to U,\ P \vee S \Rightarrow U$

(ii) Show that the premises "one student in this class knows how to write programs in JAVA"and "Every one who knows how to write programs in JAVA can get a high- paying jop" imply the conclusion "some one in this class can get high paying job CO1 -App (8)

17. (a) (i) Using mathematical induction show that $n^3 + (n+1)^3 + (n+2)^3$ is a multiple of 9. CO2 -App (8)

(ii) Solve $a_n - 4a_{n-1} + 4a_{n-2} = 2^n, a_0 = 1, a_1 = 1$. CO2 -App (8)

Or

(b) (i) Calculate the number of positive integers not exceeding 1200 that are divisible by 2 ,3 ,5 or by 7 CO2 -App (8)

(ii) Using generating functions Solve $a_n = 3a_{n-1} + 5^n, a_0 = 4$ CO2 -App (8)

18. (a) (i) Let G be a finite group of order 'n' and H be any subgroup of G . Then Show that the order of H divides the order of G. (i.e) O(H) / O(G) CO3- App (8)

(ii) Show that $(Q^+, *)$ is ab abelian Group. Where * defined as a*b=$\frac{ab}{2}$ where a, b$\in Q^+$ CO3- App (8)

Or

(b) S = Q × Q , such that binary operation defined by $(a,b)*(x,y) = (ax, ay+b)$ CO3- App (16)

(i) Prove that (S, *) is a semi group

(ii). Is it commutative and calculate the value of (2,4)*(1,5)

(iii) Find the identity Element

(iv) Find the inverse of (2,3)*(8,6) and (0,2)*(3,5)

19. (a) (i) If $y = a\cos(\log x) + b\sin(\log x)$ Show that $x^2 y_2 + xy_1 + y = 0$ CO4-App (8)

(ii) Compute the value of a,b,c if $\lim_{x\to 0} \frac{ae^x - be^{-x} - cx}{x - \sin x} = 4$ CO4-App (8)

Or

(b) (i) Compute $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\tan x}}$ CO4-App (8)

(ii) Evaluate $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$ CO4-App (8)

20. (a) (i) Find the volume of the tetrahedron bounded by $6x + 4y + z = 12$, x = 0, y = 0, z = 0. CO5- App (8)

(ii) Compute the area between the parabola $y^2 = x$ and $x^2 = y$ CO5- App (8)

Or

(b) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$ CO5- App (8)

(ii) Change the order of integration and hence evaluate CO5- App (8)

$$\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xy\,dx\,dy$$