A **Reg. No. :** | | | | | | | | | | | | |

Question Paper Code: U2M03

B.E./B.Tech. DEGREE EXAMINATION, NOV 2022

Second Semester

Computer Science and Engineering

21UMA203- Differential Equations and Complex analysis

(Regulations 2021)

(Common to information technology)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The Particular integral of $y'' + 4y' + 4y = 0$ is _______ CO1-Ap

   (a) xe^{-2x} (b) xe^{2x} (c) $x^2 e^{2x}$ (d) 0

2. The complementary function of $(4D^2 - 3D - 1)y = 2 \sin 2x$ is ______ CO6-U

   (a) $Ae^{x} + Be^{-\frac{x}{4}}$ (b) $Ae^{-x}+Be^{5x}$ (c) $(A+Bx)e^{2x}$ (d) $Ae^{x}+Be^{4x}$

3. IF $\vec{F}$ is Irrotational then $\nabla X\vec{F} =$ CO2-U

   (a) 1 (b) 2 (c) 0 (d) 3

4. Divergence of vector $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at (1, 2, -3) is ______ CO2-App

   (a) 8 (b)4 (c)-3 (d) 0

5. The critical point of the transformation $w = z + \frac{1}{z}$ are _____ CO3- App

   a) ± 1 b) ± 2 c) $\pm i$ d) $-i$

6. The function $f(z) = \frac{1}{z^2+4}$ is not analytic at $z =$ _____ . CO3- App

   (a) 2 b) -2 c)2i d)$\pm 2i$

7. Simple pole is a pole of order ________ CO6-U

   (a) 1 (b) 4 (c) 3 (d) -4

8. $\int_C \frac{e^z}{z-2} dz$ where C is the unit circle with centre as origin is ________ CO4-App

(a) 0 (d) 1 (c) 2 (d) π

9. The PDE obtained from z = (x+a)(y+b) is __. CO5-App

(a) 3z = px + qy (b) py - qx = 0 (c) z = pq (d) px+qy = 0

10. The one dimensional wave equations require _______ boundary conditions CO5-U

(a) 4 (b) 3 (c) 2 (d) 1

PART – B (5 x 2= 10Marks)

11. Calculate the Particular integral of $(D^2 + 3D + 2)y = \sin 3x$ CO1-App

12. Compute $\nabla\varphi$, if $\varphi = x^2 + y^2 + z^2$ at (1, -1, 1). CO2-App

13. Prove that $u = e^x \cos y$ is harmonic function CO3-App

14. Using Cauchy's integral formula , Evaluate $\int_c \frac{z}{z-2} dz$ where C is $|z| = 1$ CO4-App

15. Write the three Possible solutions of the one dimensional wave equations CO5-U

PART – C (5 x 16= 80Marks)

16. (a) (i) Solve $(D^2 - 4D + 3)y = e^{3x} + x^2$ CO1-App (8)

(ii) Using method of variation of parameters solve $(D^2 + a^2)y = \tan ax$ CO1- App (8)

Or

(b) (i) Solve $(x^2D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$ CO1- App (8)

(ii) A colony of bacteria of growing exponentially. At time t=0 it has 10 bacteria in it and at time t = 4 it has 2000. At what time will it have 100,000 bacteria? CO1- App (8)

17. (a) Verify Stokes theorem for a vector field defined by $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ in the rectangular region in the XOY plane bounded by the lines $x = \pm a, y = 0, and \ y = b$. CO2-App (16)

Or

(b) Verify Gauss divergence theorem for the vector function $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0\ and x = 1, y = 1, z = 1$ — CO2 -App (16)

18. (a) (i) Determine the analytic function for which — CO3-App (8)

$$U - V = \frac{\sin 2x}{\cosh 2y - \cos 2x}$$

(ii) Determine the image of $|z - 2i| = 2$ under the transformation — CO3-App (8)

$$w = \frac{1}{z}$$

Or

(b) (i) If f(z)=u+iv is an analytic function then Prove that — CO3-App (8)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f^1(z)|^2$$

(ii) Determine the bilinear transformation which maps z = 1,i,-1 respectively onto w = i,0,-i — CO3-App (8)

19. (a) (i) Using Cauchy's integral formula Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle |z|= 3 — CO4-App (8)

(ii) Evaluate f(z) = $\frac{7z - 2}{z(z+1)(z-2)}$ in Laurent's series valid in the region $1 < |z + 1| < 3$ — CO4-App (8)

Or

(b) Using Contour integration Prove that — CO4-App (16)

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a + b}\quad a > b > 0$$

20. (a) (i) Solve : $Z = px + qy + \sqrt{(1 + p^2 + q^2)}$ — CO5-App (8)

(ii) Solve : $x(y - z)p + y(z - x)q = z(x - y)$ — CO5- App (8)

Or

(b) A tightly String with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If its set vibrating giving each point at velocity $\lambda(lx - x^2)$. Determine the displacement function y(x,t). — CO5- App (16)